The Superman Punch

Once the fighter launches himself into the air, the trajectory of his shoulder follows a parabola. Suppose the angle between the launching direction and the flat ground is θ, and the launching speed is v. The vertical component of the speed is $v\sin\theta$ and the horizontal component of the speed is $v\cos\theta$.

Let (x, y) represent the coordinates of the shoulder and let the initial position of the shoulder be (0, 0). According to Newton's Laws (neglecting any air resistance),

$$\begin{cases} \ddot{x} = 0 \\ \ddot{y} = -g \end{cases}$$

Where g is the acceleration due to earth's gravity. Integrating once we have

$$\begin{cases} \dot{x} = (v\cos\theta) \\ \dot{y} = -gt + (v\sin\theta) \end{cases} \qquad (1)$$

Integrating once more we have

$$\begin{cases} x = (v\cos\theta)t \\ y = -0.5gt^2 + (v\sin\theta)t \end{cases} \qquad (2)$$

Denote the speed of the shoulder by u.

$$u^2 = (\dot{x})^2 + (\dot{y})^2 = v^2 - (2gv\sin\theta)t + g^2t^2$$

This is a quadratic function with its graph opening upward. Its domain is $\left[0, \frac{2v\sin\theta}{g}\right]$. The domain is obtained by solving $y = -0.5gt^2 + (v\sin\theta)t = 0$.

The maximum value of $u^2 = v^2$ occurs at $t = 0$ and $t = \frac{2v\sin\theta}{g}$ (launching and landing).

The minimum value of $u^2 = v^2\cos^2\theta$ occurs at $t = \frac{v\sin\theta}{g}$ (at the highest point of the parabola).

Eliminating the parameter t from (2) yields:

$$y = -\frac{g}{2v^2\cos^2\theta}x^2 + (\tan\theta)x$$

Its derivative, i.e., the slope of the tangent line of the trajectory, is

$$\frac{dy}{dx} = -\frac{g}{v^2 \cos^2 \theta} x + \tan\theta$$

Which is also the tangent function of the angle α between the tangent line and the horizontal line ,

$$\tan\alpha = -\frac{g}{v^2 \cos^2 \theta} x + \tan\theta$$

Or

$$\alpha = \tan^{-1}\left(-\frac{g}{v^2 \cos^2 \theta} x + \tan\theta\right) = \tan^{-1}\left(-\frac{g}{v\cos\theta} t + \tan\theta\right)$$

At $t = \frac{v\sin\theta}{g}$ (when speed is min), $\alpha = \tan^{-1}(0) = 0$.

At $t = \frac{2v\sin\theta}{g}$ (landing, when speed is max), $\alpha = \tan^{-1}(-\tan\theta) = -\theta$.

At $t = 0$ (launching, when speed is max), $\alpha = \tan^{-1}(\tan\theta) = \theta$.