

A Cognitive Approach to Learning Order of Operations: Lessons from Language Processing

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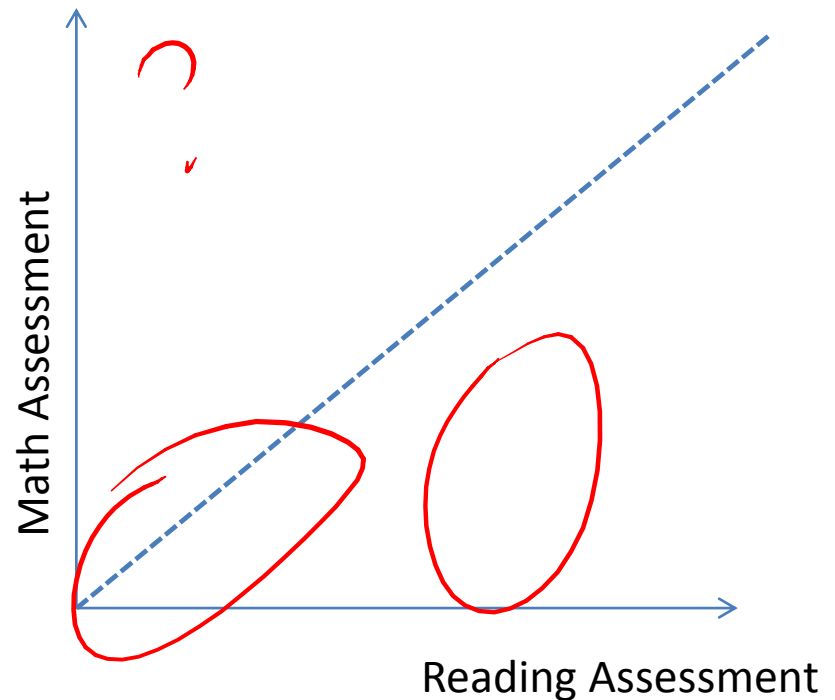
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Reading and Math Placements in Community Colleges

English	Tot/FY	% placed
CSKLS 312	109	1.84%
CSKLS 313	388	6.55%
ENGL 305.1,305X,306X	656	11.07%
ENGL 307,302X,100X	1140	19.23%
ENGL 100	1531	25.83%
ENGL 1A	2103	35.48%
Total English	5927	

Math	Tot/Fy	% placed
CSKLS 371	1478	24.14%
CSKLS 372	883	14.42%
MATH 151	1373	22.42%
MATH 101/155	987	16.12%
MATH 9	828	13.52%
MATH 25	95	1.55%
MATH 58	199	3.25%
MATH 1A	280	4.57%
Math Total	6123	



*Data from SRJC FY 2013-2014

Dear John

I want a man who knows what love is all about
you are generous kind thoughtful people who
are not like you admit to being useless and
inferior you have ruined me for other men I
yearn for you I have no feelings whatsoever
when we're apart I can be forever happy will
you let me be yours Amber

Dear John,

I want a man who knows what love is all about. You are generous, kind, thoughtful. People who are not like you admit to being useless and inferior. You have ruined me for other men. I yearn for you. I have no feelings whatsoever when we're apart. I can be forever happy -- will you let me be yours?

--Amber

Dear John,

I want a man who knows what love is. All about you are generous, kind, thoughtful, people, who are not like you. Admit to being useless and inferior. You have ruined me. For other men I yearn. For you I have no feelings whatsoever. When we're apart, I can be forever happy. Will you let me be?

Yours,

Amber

theredonateakettleoftenchips

The red on a teakettle often chips.

There, Don ate a kettle of ten chips.

What is -1^2 ?

Why do so many students
think this is 1?

How would we evaluate something like

$$5 - 3 * 12 + 8 \div 4 ?$$

PEMDAS

How would we evaluate something like

$$5 - 3 * 12 + 8 \div 4 ?$$

PMASED

PDASME

SADMEP etc...

Answer the following

What is:

$$2-1=$$

$$4-1=$$

$$8-7=$$

$$15-12=$$

Now think of a number between 12 and 5.

Was it 7?

Subtraction mode?
(also known as “priming” in
cognitive psychology)

$$\int \text{_____} dx$$

¿ _____ ?

ne _____ pas

In my mind this stimulates two questions.

Q1. What are the psychological/cultural/linguistic factors creating these predispositions?

Q2. What are the frequencies at which various dispositions govern the interpretive order of operations?

Methodology and Subjects

- Short quizzes containing the test items (6 each) were administered by instructors
- 8 Instructors from 3 colleges (Norco, Fullerton, SRJC) participated
- Includes nearly 1000 students
- Levels include Arithmetic, Pre-algebra, and Elementary Algebra

Test Items: Effect of Size

Facilitative Cues

Obstructive Cues

Experimental

$$-1^2$$

Experimental

$$12 \div 3 \times 2$$

Control

$$-1^2$$

Control

$$12 \div 3 \times 2$$

Effect of Size

Facilitative Cues

-1^2

-1^2

	Experi.	Control
# Correct	256	169
Sample Size	447	443
% Correct	57.3%	38.1%

$P < 0.001$

Obstructive Cues

$12 \div 3 \times 2$

$12 \div 3 \times 2$

	Experi.	Control
# Correct	133	253
Sample Size	444	454
% Correct	30.0%	55.7%

$P < 0.001$

Effect of Length

Test Items

Facilitative Cues

Experimental $\ominus 1^2$

Control $\ominus 1^2$

	Experi.	Control
# Correct	161	109
Sample Size	233	227
% Correct	69.1%	48.0%

$P < 0.001$

Effect of Prominence (size, boldness, length combined)

Facilitative Cues

	Experi.	Control
# Correct	501	322
Sample Size	784	776
% Correct	63.9%	41.5%

P < 0.001

Obstructive Cues

	Experi.	Control
# Correct	168	312
Sample Size	548	560
% Correct	30.7%	55.7%

P < 0.001

Effect of Spacing

Facilitative Cues

- -1^2 v.s. -1^2

	Experi.	Control
# Correct	294	236
Sample Size	430	436
% Correct	68.4 %	54.1 %

$P < 0.001$

Obstructive Cues

- $5 + 2 \times 4$ v.s. $5 + 2 \times 4$

	Experi.	Control
# Correct	129	135
Sample Size	214	218
% Correct	60.3 %	61.9 %

$P = 0.36$

Reading as Sentence Processing

- Each language has a **grammar** that specifies the **syntactic structure** of any sentence.

Production Rules:

[Sentence] → [Noun Phrase] [Verb Phrase]

[Noun Phrase] → [Noun]

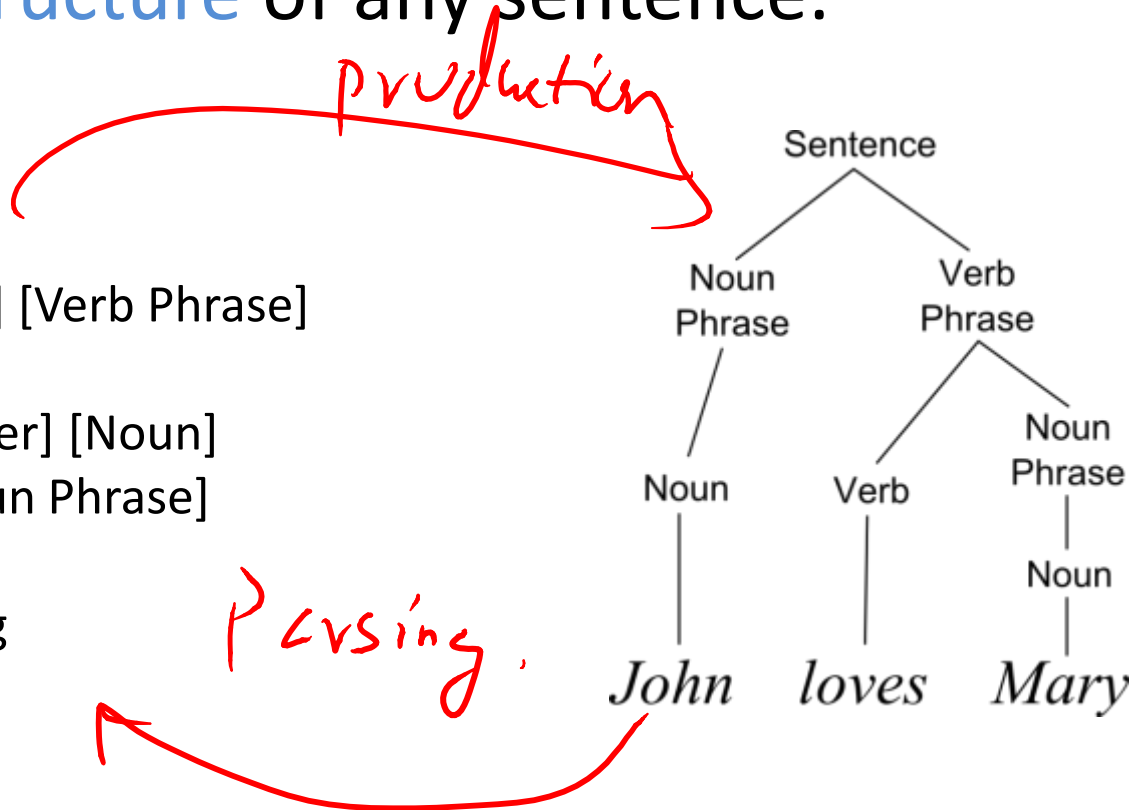
[Noun Phrase] → [Determiner] [Noun]

[Verb Phrase] → [Verb] [Noun Phrase]

[Verb] → loves

[Noun] → John | Mary | dog

[Determiner] → the



Parsing Algebraic Expressions

- A grammar that deals with the correct order of operations

[Expression] \rightarrow [Expression] + [Term]

[Expression] \rightarrow [Expression] - [Term]

[Expression] \rightarrow [Term]

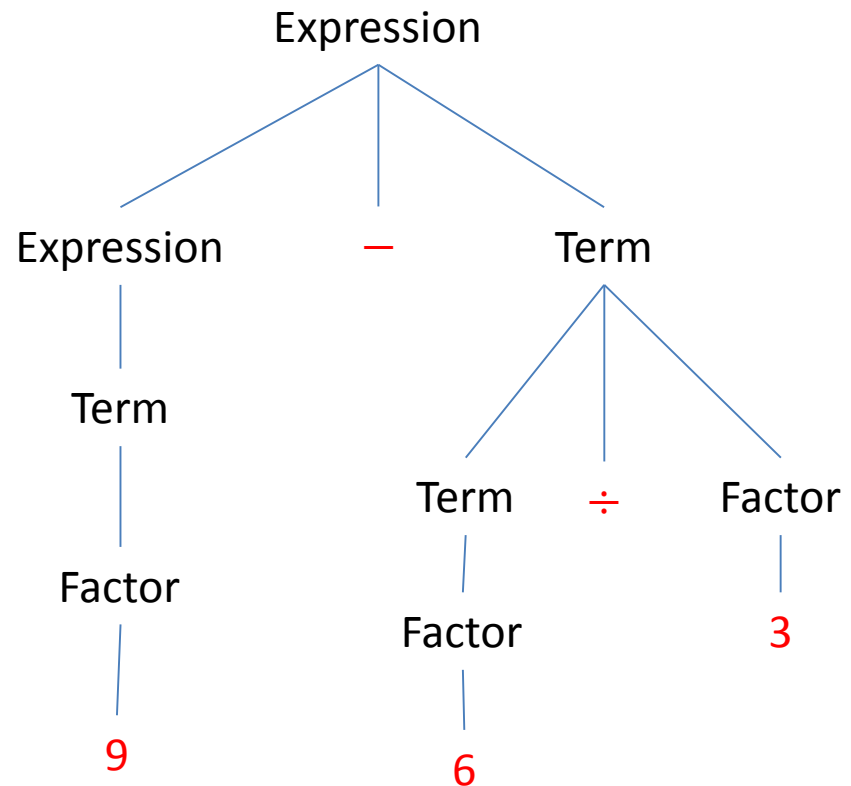
[Term] \rightarrow [Term] \div [Factor]

[Term] \rightarrow [Term] \times [Factor]

[Term] \rightarrow [Factor]

[Factor] \rightarrow 1 | 2 | ... | 9

[Factor] \rightarrow ([Expression])



Parsing Algebraic Expressions

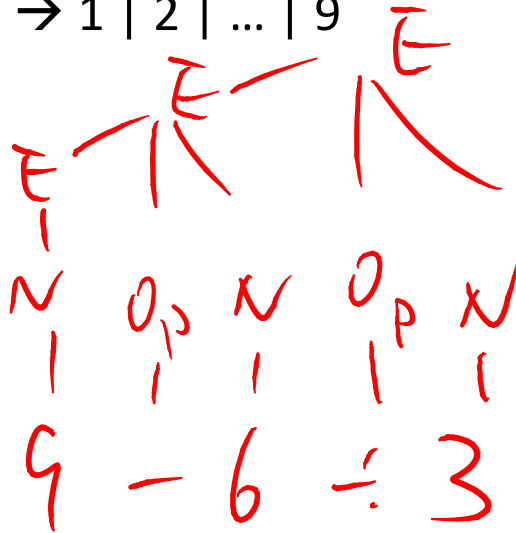
- An “intuitive” grammar that produces ambiguous parses for $9 - 6 \div 3$

[Expression] \rightarrow [Expression] [Operator] [Number]

[Expression] \rightarrow [Number]

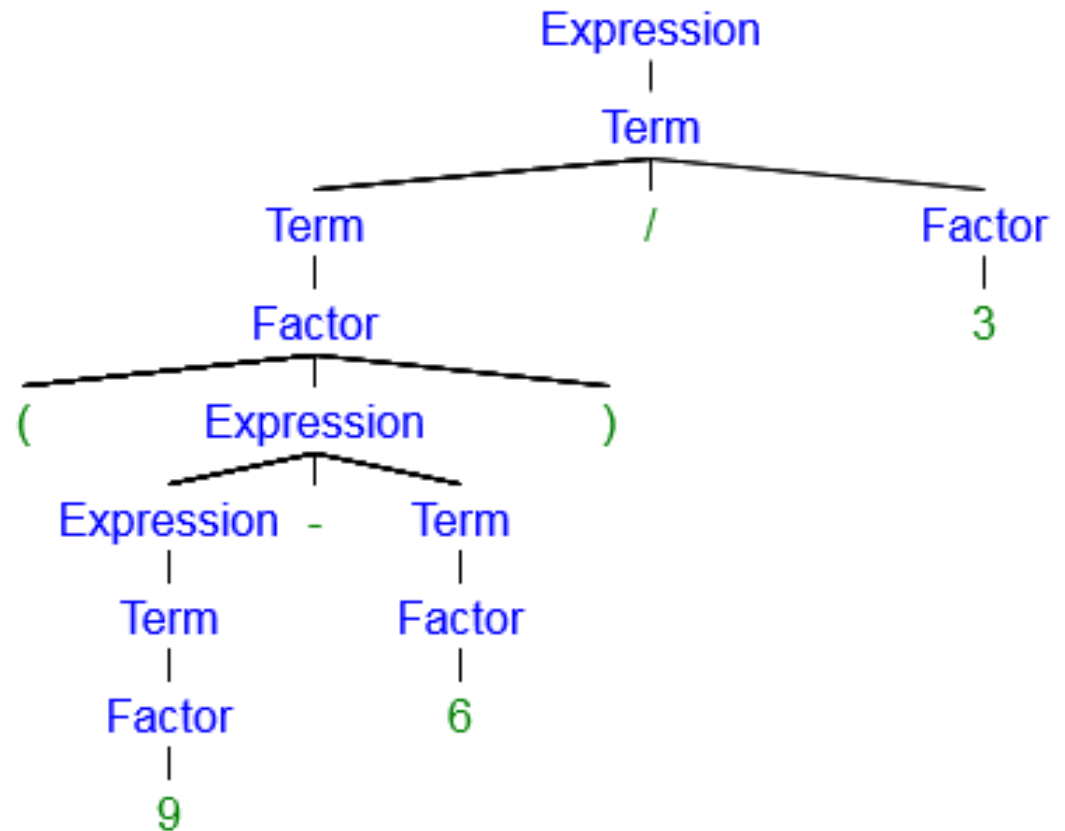
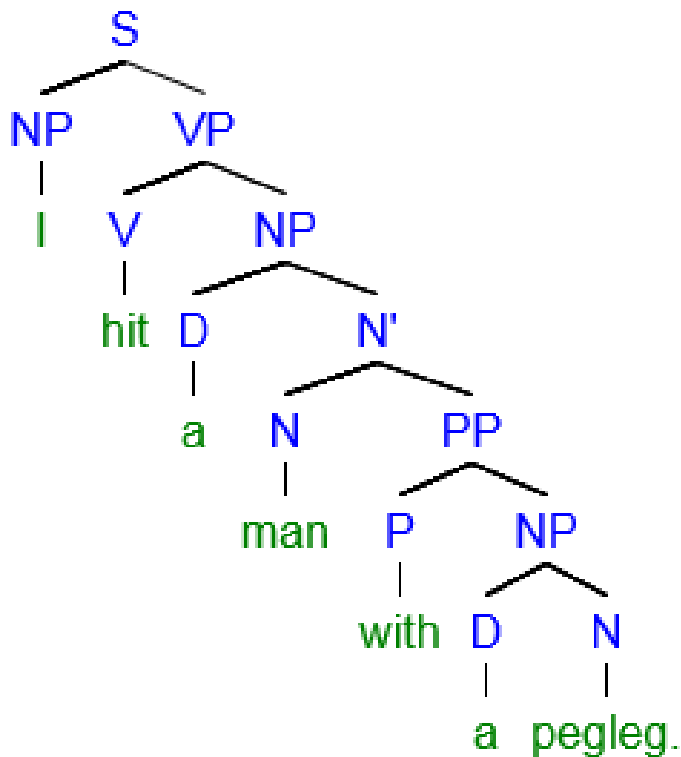
[Operator] $\rightarrow + \mid - \mid \times \mid \div$

[Number] $\rightarrow 1 \mid 2 \mid \dots \mid 9$



Are the trees real?

- We don't know. But there is plenty of evidence that people group the words first.



Psycholinguistic Research on Reading (Bever, 1992)

A strategy of chunking sentences into phrase units with these cues could be very effective, based purely on memorized features and not on meaning.

The same sentence will be easier to read if formatted as:

A strategy of chunking sentences
into phrase units with these cues
could be very effective
based purely on memorized features
and not on meaning.

How spacing affects reading text

- **Phrase-spaced:**

At Frobisher, "Capital" of Baffin Island, where the airfield built by the United States in World War II and later purchased by Canada is being expanded, Eskimos are in jet-propelled transition from the Stone Age to alarm clocks, Western movies, and hi-fi.

- **Even-spaced:**

At Frobisher, "Capital" of Baffin Island, where the airfield built by the United States in World War II and later purchased by Canada is being expanded, Eskimos are in jet-propelled transition from the Stone Age to alarm clocks, Western movies, and hi-fi.

Main findings regarding Effect of Spacing

- Cromer (1970): people with good vocabulary and reading disabilities benefit from extra spacing that shows the correct grouping of words.
- Bever (1991, 1992): Formatting the text with spacing cues improves the comprehension of **average** readers, but no effect on advanced readers.

Summary of Effect of Phrase-based Spacing

Comprehension and/or Recall Significant Studies

	Phrase-spaced	Control	Difference
Anglin & Miller (1968)	53.0	42.0	+26.0
Brozo et al. (1983)	17.5	16.8	+ 4.0
Cromer (1970)	59.0	45.0	+14.0
	50.4	22.4	+28.0
Gerrell & Mason (1983)	72.3	66.0	+ 9.5
Graf & Torrey (1966)	54.7	4.18	+30.9
Mason & Kendall (1979)	62.0	52.0	+19.0
Negin (1982)	9.3	8.1	+16.1
North & Jenkins (1951)	494.0	448.0	+10.9
O'Shea & Sindelar (1983)	68.1	63.4	+ 7.4
Stevens (1981)	65.0	54.0	+11.0

Summary and Interpretation of Our Results

- Dev Ed students' grouping strategies for algebraic expressions is impacted by presence of cues in the stimuli
- Such influence parallels the reading research in the past 40+ years
- Reading and mathematics likely share a deep common mechanism for processing strings of symbols

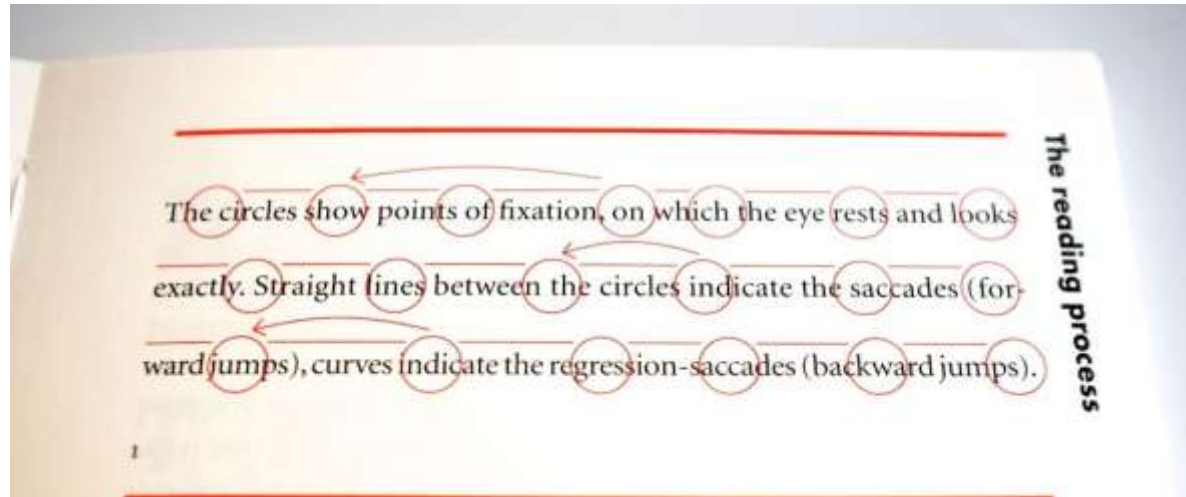
Implications for the Classroom

- Should formatting be used to facilitate the learning of order of operations?
 - Esp. for students who are good readers but have genuinely difficulty in reading algebraic expressions
- Should we be teaching grouping strategies/trees in addition to PEMDAS?

Future Explorations

- New task / design
 - Measuring speed as well as accuracy.
 - Within-subject design to minimize the effect of test items
- Automatic formatting of teaching materials
- Using eye-tracking to identify the rapid eye movements during reading algebraic expressions

Eye Movement Patterns During Reading



References

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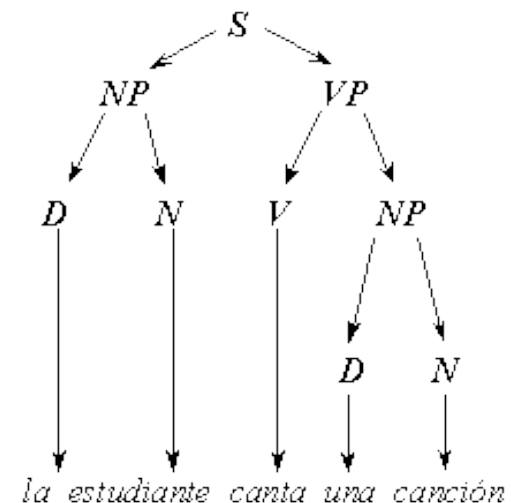
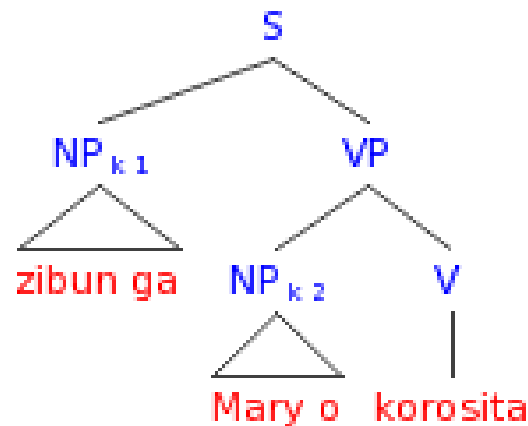
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Arguments for Linguistic Structure

- 1) *Colorless green ideas sleep furiously*
- 2) *Furiously sleep ideas green colorless.*



Acoustic Cues for Sentence Processing

