Math to Math Resuscitation

Joe Vasta

Cuesta College

CMC^3

Monterey

2013

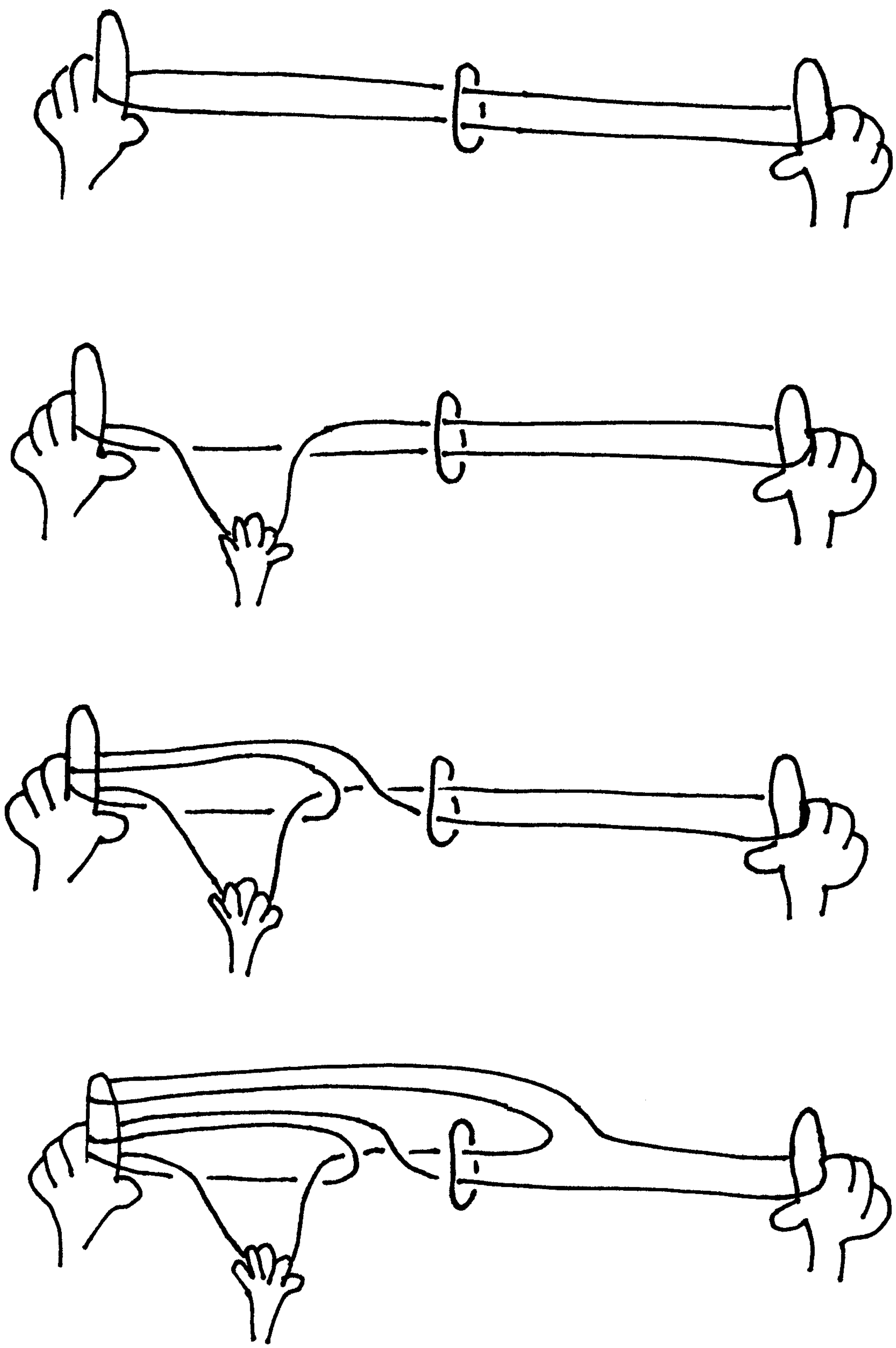
Precalculus

version $\lfloor \pi \rfloor$

Carl Stitz
&
Jeff Zeager

free textbook

Take the Ring from the String



Let go of the string, and take the ring.

Formula for the Fibonacci Numbers

Odd number sequence

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, . . .

recursive formula $a_n = a_{n-1} + 2 \qquad a_1 = 1$

explicit formula $a_n = 2n - 1$

Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .

recursive formula $f_n = f_{n-1} + f_{n-2} \qquad f_1 = f_2 = 1$

explicit formula $f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

Generating a Fibonacci Function

(1) Recursive Formula:

$$f_{n+2} = f_{n+1} + f_n \qquad f_0 = 0,\ f_1 = 1$$

(2) Generating Function:

$$G(x) = \sum_{n=0}^{\infty} f_n x^n = f_0 + f_1 x + f_2 x^2 + \cdots$$

(3) Power Series Equation:

$$\sum_{n=0}^{\infty} f_{n+2} x^n = \sum_{n=0}^{\infty} f_{n+1} x^n + \sum_{n=0}^{\infty} f_n x^n$$

(4) Use some algebra:

$$\frac{1}{x^2}\left(G(x) - x\right) = \frac{1}{x} G(x) + G(x)$$

(5) Solve for $G(x)$:

$$G(x) = \frac{-x}{x^2 + x - 1}$$

(6) Roots of $x^2 + x - 1$:

$$c = \frac{-1+\sqrt{5}}{2}, \quad d = \frac{-1-\sqrt{5}}{2}$$

(7) Calculate reciprocals:

$$\frac{1}{c} = \frac{1+\sqrt{5}}{2}, \quad \frac{1}{d} = \frac{1-\sqrt{5}}{2}$$

(to be used later)

(8) Partial Fractions :

$$G(x) = \frac{1}{\sqrt{5}}\left(\frac{-c}{x-c}\right) + \frac{1}{\sqrt{5}}\left(\frac{d}{x-d}\right)$$

(use $c - d = \sqrt{5}$)

(9) Use some algebra:

$$G(x) = \frac{1}{\sqrt{5}}\left(\frac{1}{1-\frac{x}{c}}\right) - \frac{1}{\sqrt{5}}\left(\frac{1}{1-\frac{x}{d}}\right)$$

(10) Use some calculus:

$$G(x) = \sum_{n=0}^{\infty}\left[\frac{1}{\sqrt{5}}\left(\frac{1}{c}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1}{d}\right)^n\right] x^n$$

(11) Explicit Formula:

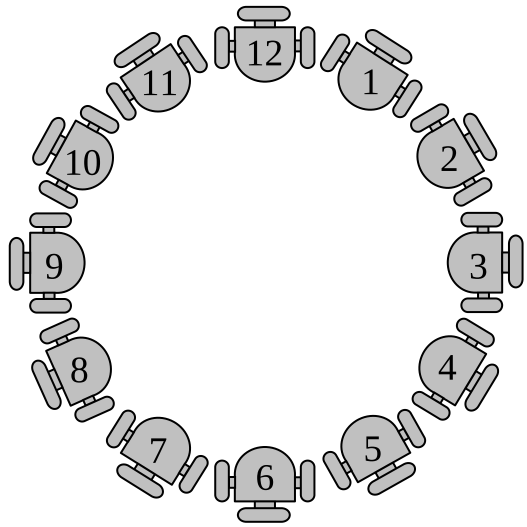
$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1}{c}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1}{d}\right)^n$$

(12) Use reciprocal calculation:

$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Formula for a Counting Problem

A mean math teacher forgets to write a final for his class. When his twelve students come to take the final, they find twelve chairs in a circle with their teacher in the middle. The chairs are numbered one through twelve, and the teacher tells the students to sit in the chairs.



The teacher tells student #1 to leave. After #1 leaves, he skips a student and tells #3 to leave. After #3 leaves, he skips a student and tells #5 to leave. He continues in this fashion until there is one student left. The last student is the survivor and passes the class. The rest of the students fail the class. Which student is the survivor?

8

If there were one hundred students, then which student would be the survivor?

72

The survivor function outputs the survivor. The input is the number of students.

$$S(x) = 2x - 2^{\lceil \log_2 x \rceil}$$

(This problem is based on the famous Josephus Problem.)

Ripping Paper & Exponents

Take one piece of paper. Rip it in half. Put the pieces of paper together, and rip them in half. Put those pieces of paper together, and rip them in half. How many pieces of paper would there be after 50 rips?

____

Rips	Pieces of Paper
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$
5	$32 = 2^5$
6	$64 = 2^6$

The exponent represents the number of times the paper was ripped.

Ripping Paper & Exponents

Answer: $2^{50} = 1,125,899,906,842,624$ pieces of paper

(*one quadrillion, one hundred twenty-five trillion, eight hundred ninety-nine billion, nine hundred six million, eight hundred forty-two thousand, six hundred twenty-four*)

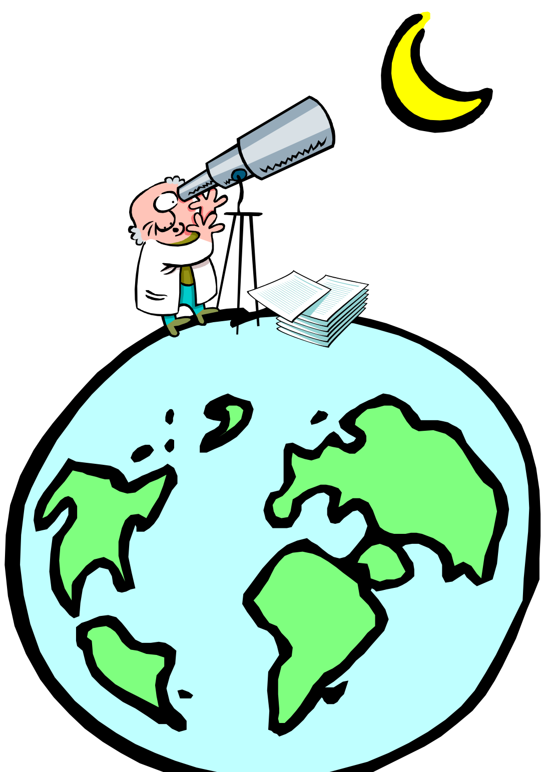
Assuming the thickness of the paper is 0.001 inch, we can stack the pieces of paper and calculate the height of the pile.

height = (0.001)(inch) × (1,125,899,906,842,624) ≈ 1,125,899,906,843 inches

$$\text{height} = \left(1{,}125{,}899{,}906{,}843 \text{ inches}\right) \times \left(\frac{1 \text{ foot}}{12 \text{ inches}}\right) \times \left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)$$
$$\approx 17{,}769{,}885 \text{ miles}$$

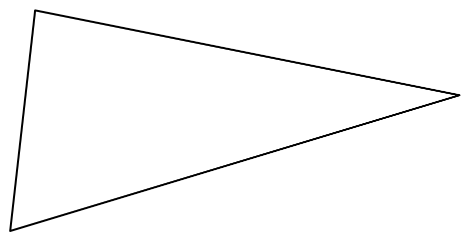
The average distance from the Earth to the moon is approximately 238,855 miles.

Theoretically, after ripping the paper 50 times, we could make 74 stacks of paper, each stack piled from the Earth to the moon!!!!!

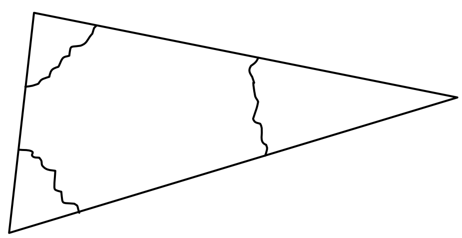


Ripping Triangles

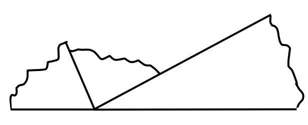
I. Find a triangle.



II. Rip the corners off the triangle.



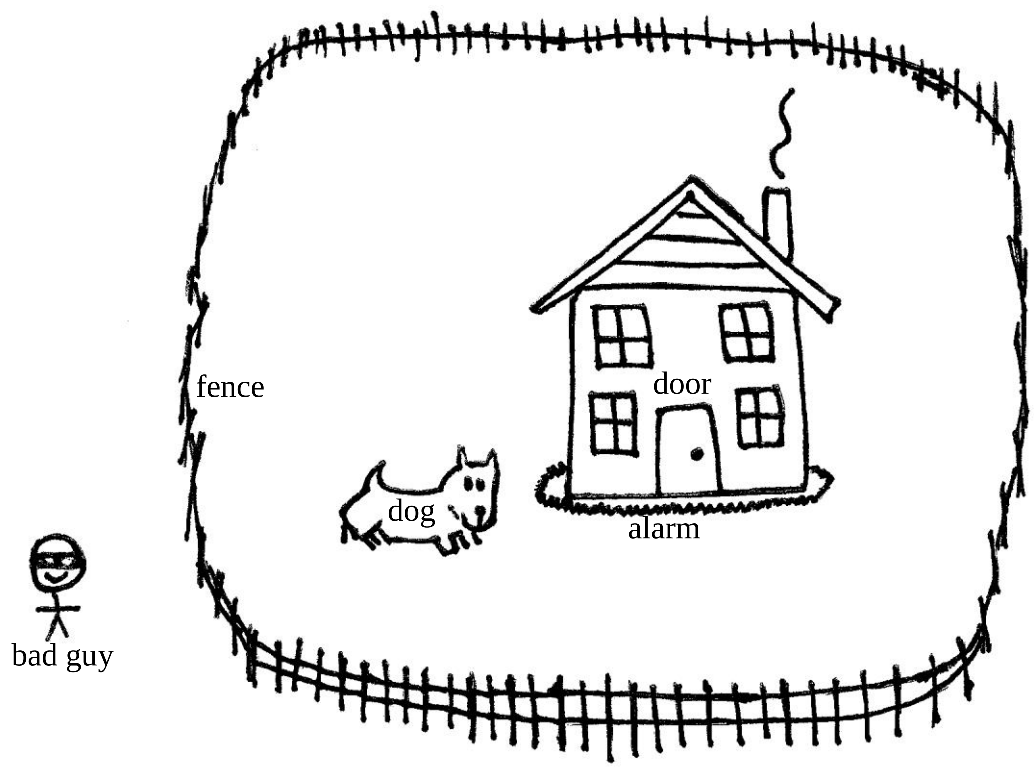
III. Arrange the corners to create a straight angle.



The Chain Rule

Find the derivative: $y = \sqrt{\sin e^{(5x+3)}}$

The chain rule is like breaking into the house. You must get past the fence, the dog, the alarm, and the door (in that order).



$$y = \sqrt{\sin e^{(5x+3)}} = \left(\sin\left(e^{(5x+3)}\right)\right)^{1/2} = \left(\text{dog}\left(\text{alarm}\left(\text{door}\right)\right)\right)^{\text{fence}}$$

$$y' = \frac{1}{2}\left(\sin\left(e^{(5x+3)}\right)\right)^{-1/2} \cdot \cos\left(e^{(5x+3)}\right) \cdot e^{(5x+3)} \cdot 5$$

$$y' = \frac{5e^{(5x+3)} \cos e^{(5x+3)}}{2\sqrt{\sin e^{(5x+3)}}}$$

Which is Greater: e^{π} or π^{e} ?

We will do this problem without using a calculator.

STEP 1: Let $f(x) = x^{\frac{1}{x}}$.

Let's investigate the graph of f on the interval $(0, \infty)$.

STEP 2: $f'(x) = \dfrac{x^{\frac{1}{x}}(1 - \ln x)}{x^2}$ (using logarithmic differentiation)

STEP 3: We see that *e* is a critical number.

STEP 4: Using the 1st Derivative Test, we see that *e* is a local max.

STEP 5: So,

$$f(e) > f(\pi)$$

$$e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$$

$$\left(e^{\frac{1}{e}}\right)^{\pi} > \left(\pi^{\frac{1}{\pi}}\right)^{\pi}$$

$$e^{\frac{\pi}{e}} > \pi$$

$$\left(e^{\frac{\pi}{e}}\right)^{e} > (\pi)^{e}$$

$$e^{\pi} > \pi^{e}$$

Try	3.
a	1
coke,	4
a	1
large	5
hamburger,	9
or	2
french	6
fries	5
and	3
you'll	5
remember	8
seventeen	9
numbers	7
beginning	9
the	3
pi.	2
You	3
shouldn't	8
make	4
trying	6
to	2
digest	6
food	4
the	3
one	3
mnemonic	8
for	3
pi.	2
However,	7
digesting	9
fries,	5
hamburgers,	0
or	2
unwanted	8
calories	8
gets	4
a	1
sensitive	9
stomach,	7
a	1
bigger	6
waistline,	9
and	3
unhealthy	9
movements.	9
-Joe	3
Charles	7
Vasta	5

To	2.
impress	7
a	1
fraction	8
of	2
students	8
I	1
remember	8
to	2
numerate	8
this	4
short	5
statement	9
connecting	0
some	4
words	5
to	2
the	3
decay	5
and	3
growth	6
discussion.	0
If	2
calculus	8
creates	7
fear	4
causing	7
a	1
big	3
child	5
to	2
simply	6
freeze	6
up,	2
then	4
obviously	9
college	7
faculty	7

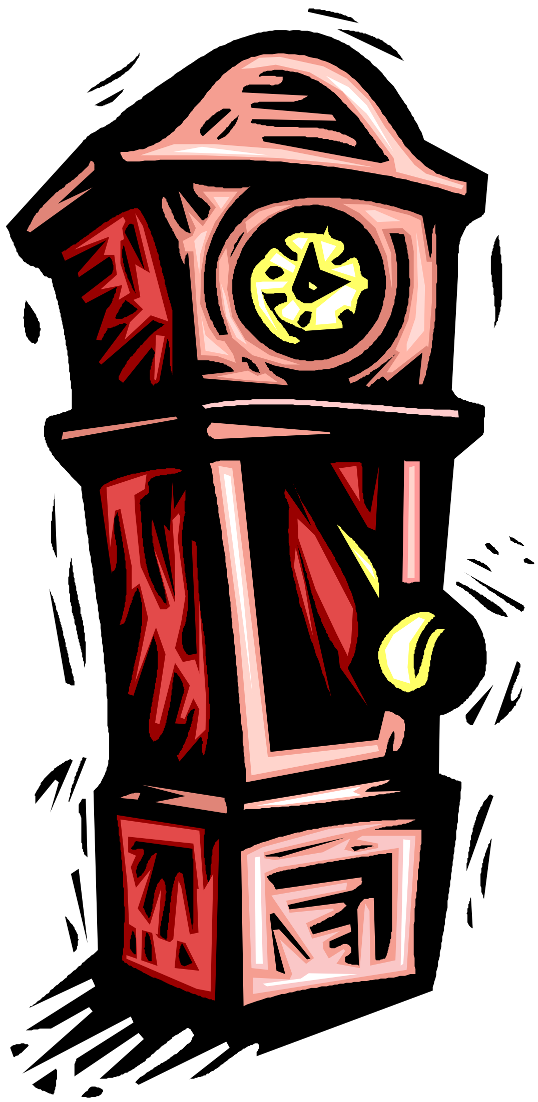
could	5
attempt	7
to	2
show	4
greater	7
discipline	0
involving	9
the	3
coffee	6
addiction.	9
Logically,	9
realizing	9
their	5
beverages	9
alter	5
actions,	7
some	4
lecturers	9
should	6
surely	6
terminate	9
coffee	6
binging	7
before	6
it	2
reduces	7
dignity	7
of	2
math	4
professors	0
without	7
coffee	6
issues	6
and	3
addictions.	0
Joe	3
Vasta	5

A Quadratic Story

A negative boy couldn't decide
on a radical party.

The boy was a square.

He lost 4 awesome chicks.



And it was all over by 2 a.m.

Soap Story

Factor.

$$a^3 + b^3$$

Alice and Bob met in college.

$$(a \quad b)(\qquad\qquad\qquad)$$

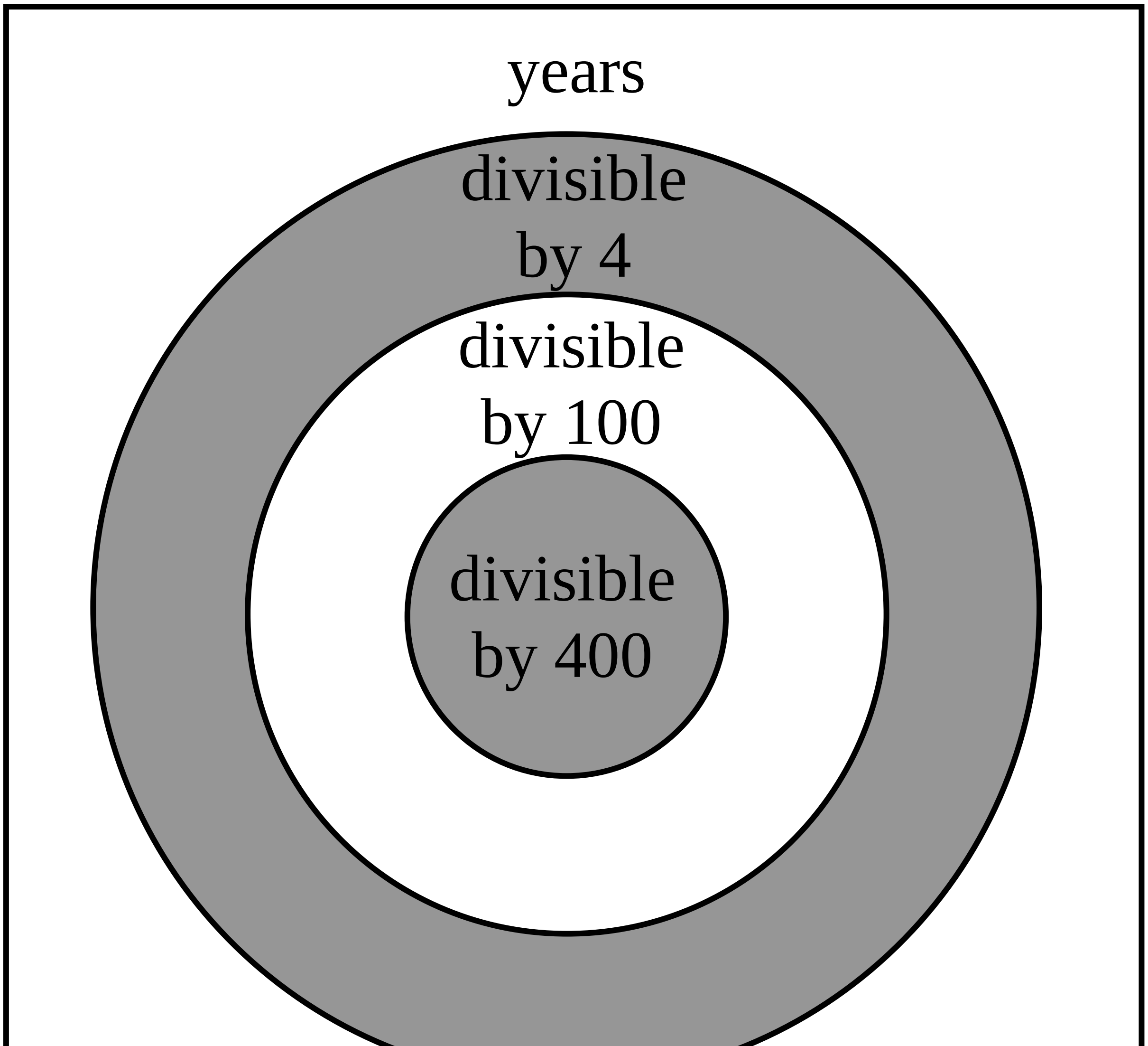
They earned their degrees,
got married, and had a kid.

$$(a \quad b)\left(a^2 \quad ab \quad b^2\right)$$

The family regularly uses SOAP.
(**S**ame, **O**pposite, **A**lways **P**ositive)

$$\left(a+b\right)\left(a^2-ab+b^2\right)$$

Leap Years



Sicherman Dice

Using natural numbers only, can you renumber the faces of a pair of 6-sided dice so that their sums have the same distribution as the sums from a standard pair of dice?
The outcomes of a pair of standard dice are shown below.

1 1					
1 2	2 1				
1 3	2 2	1 3			
1 4	2 3	3 2	4 1		
1 5	2 4	3 3	4 2	5 1	
1 6	2 5	3 4	4 3	5 2	6 1
2 6	3 5	4 4	5 3	6 2	
3 6	4 5	5 4	6 3		
4 6	5 5	6 4			
5 6	6 5				
6 6					

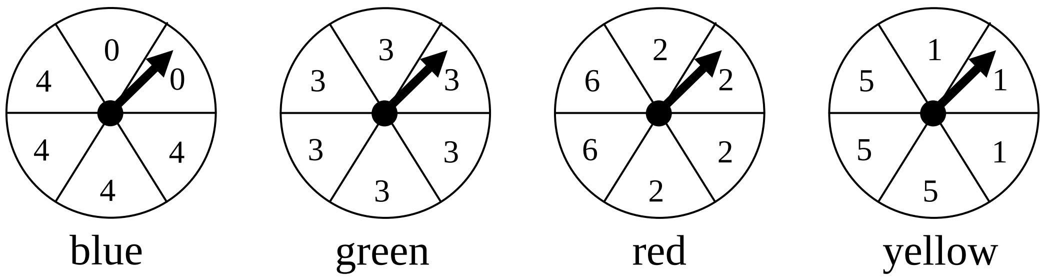
Sicherman Dice

There is a unique solution.
The first die has 1, 2, 2, 3, 3, and 4. The second die has 1, 3, 4, 5, 6, and 8.
These two dice are called Sicherman dice.
The outcomes of a pair of Sicherman dice are shown below.

1 1					
2 1	2 1				
1 3	3 1	3 1			
1 4	2 3	2 3	4 1		
1 5	2 4	2 4	3 3	3 3	
1 6	2 5	2 5	3 4	3 4	4 3
2 6	2 6	3 5	3 5	4 4	
1 8	3 6	3 6	4 5		
2 8	2 8	4 6			
3 8	3 8				
4 8					

Probability and the Spinner Game

You and a friend will each choose one spinner and spin it. The higher number wins.



Do you choose first or second? Which spinner do you choose?

Let > be the relation "has a better chance of winning than."

Blue vs. Green

B\G	3	3	3	3	3	3
0	G	G	G	G	G	G
0	G	G	G	G	G	G
4	B	B	B	B	B	B
4	B	B	B	B	B	B
4	B	B	B	B	B	B
4	B	B	B	B	B	B

Blue wins ⅔ of the time. B > G.

Green vs. Red

G\R	2	2	2	2	6	6
3	G	G	G	G	R	R
3	G	G	G	G	R	R
3	G	G	G	G	R	R
3	G	G	G	G	R	R
3	G	G	G	G	R	R
3	G	G	G	G	R	R

Green wins ⅔ of the time. G > R

Red vs. Yellow

R\Y	1	1	1	5	5	5
2	R	R	R	Y	Y	Y
2	R	R	R	Y	Y	Y
2	R	R	R	Y	Y	Y
2	R	R	R	Y	Y	Y
6	R	R	R	R	R	R
6	R	R	R	R	R	R

Red wins ⅔ of the time. R > Y

Yellow vs. Blue

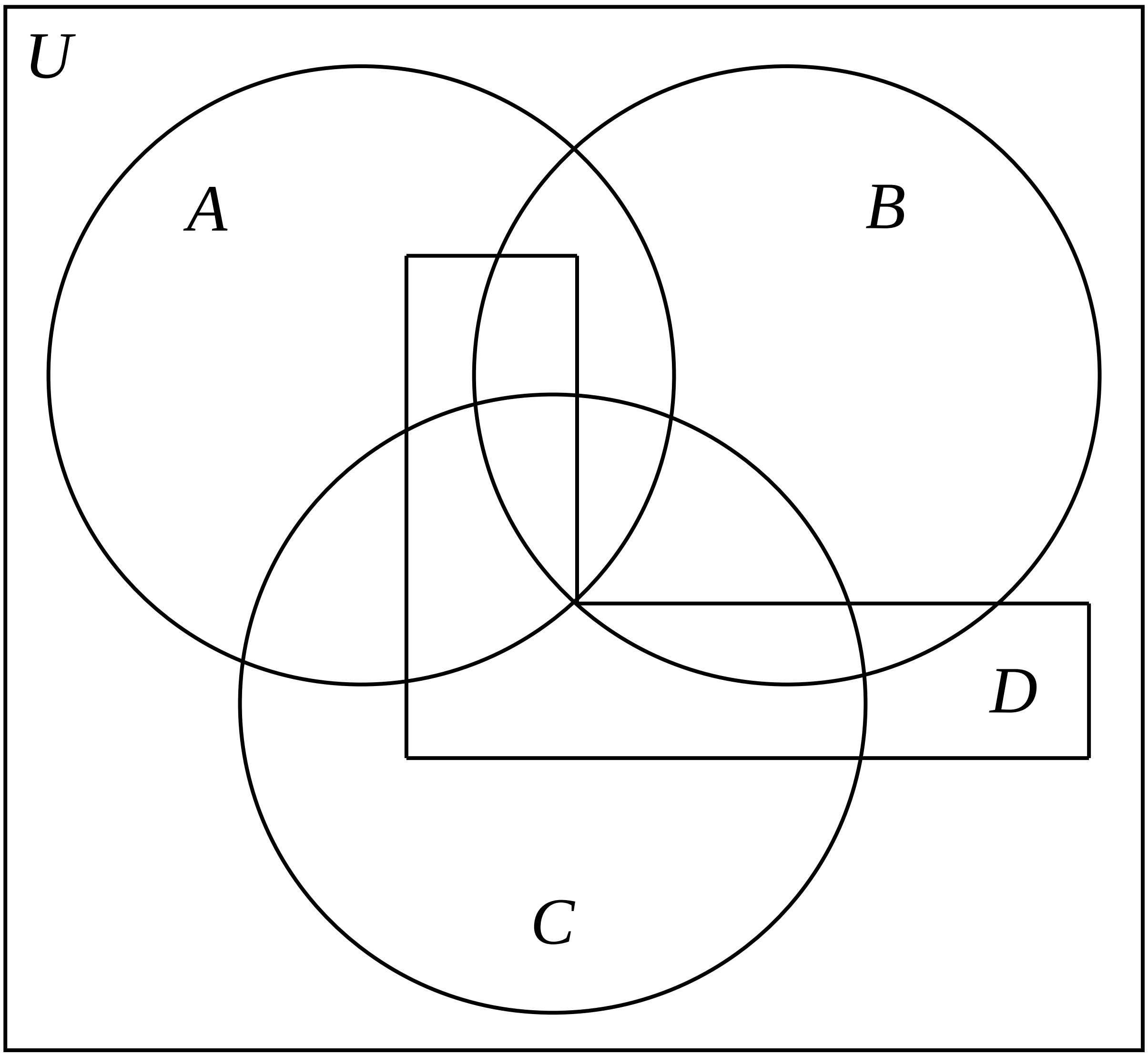
Y\B	0	0	4	4	4	4
1	Y	Y	B	B	B	B
1	Y	Y	B	B	B	B
1	Y	Y	B	B	B	B
5	Y	Y	Y	Y	Y	Y
5	Y	Y	Y	Y	Y	Y
5	Y	Y	Y	Y	Y	Y

Yellow wins ⅔ of the time. Y > B

So, B > G > R > Y > B. This means that > is a nontransitive relation.

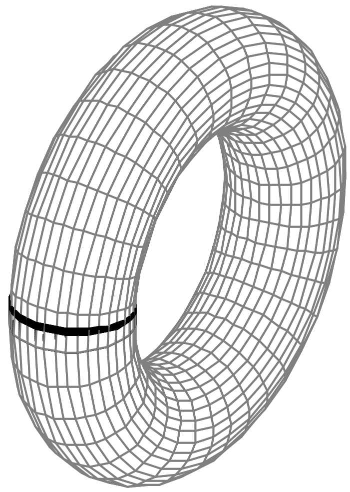
The Strategy: You let your friend choose first. You pick the color directly to the left of the chosen color in the following statement: B > G > R > Y > B. Your probability of winning with this strategy is 2/3.
These spinners are modeled from the famous Efron dice.

Venn Diagram for Four Sets

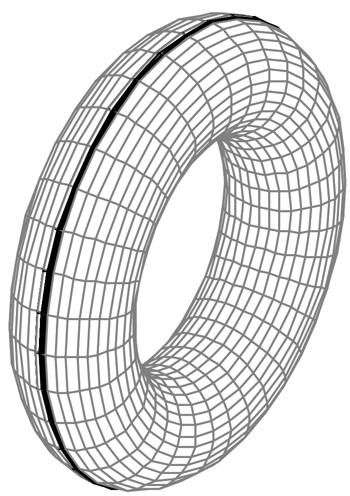


The Torus

Start with a transparent inner tube with a loop painted on it.



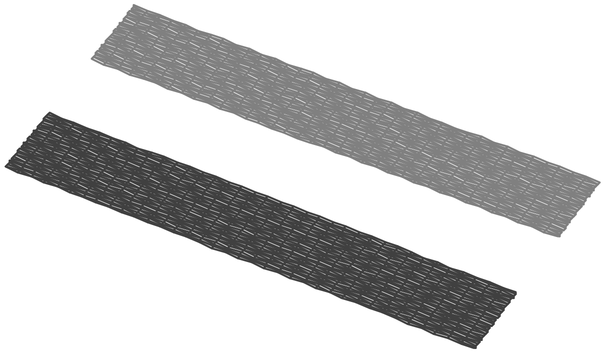
This figure is called a "torus." We will now puncture the torus. This puncture is called a "hole." Turn the torus inside out by pushing it through the hole. You might be surprised to see that the loop has a different configuration.



To actually see this work, get a square piece of cloth with stripes on both sides. Sew it together to make a torus. Cut a hole in this torus, and you will have a working model. Most people will be surprised to see this property of the torus.

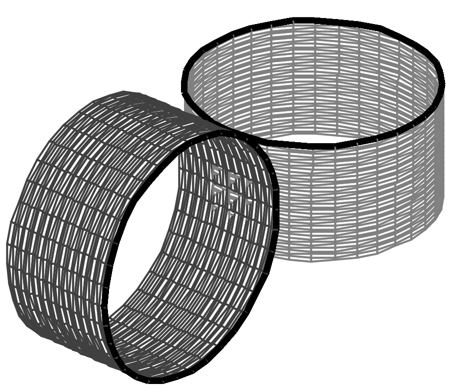
Two Loops

1. Start with two strips of paper.



2. Create two loops using tape.

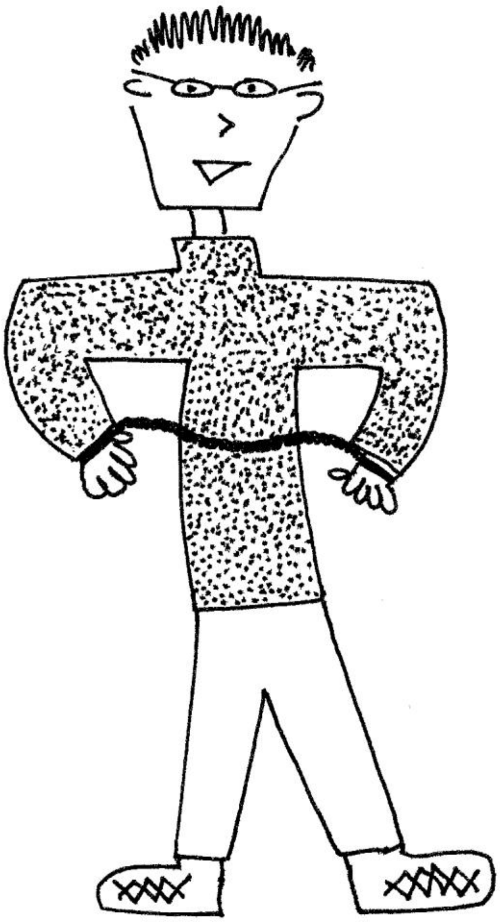
3. Tape the two loops together as shown in the picture below.



4. Cut each loop down the center. What do you end up with?

All Tied Up with Topology

Suppose your hands are tied up, and you are wearing a sweatshirt. Can you turn your sweatshirt inside out without taking the rope off of your hands?



Solution:

Step 1: Take the sweatshirt off so that it is hanging on the rope.

Step 2: Find a sleeve hole and push the sweatshirt through the sleeve hole. This is done while the sweatshirt is on the rope.

Step 3: Now put the sweatshirt back on. To do this last step, find the head hole and put your head in it. Work your arms through the sleeves. You will be wearing your sweatshirt inside out!

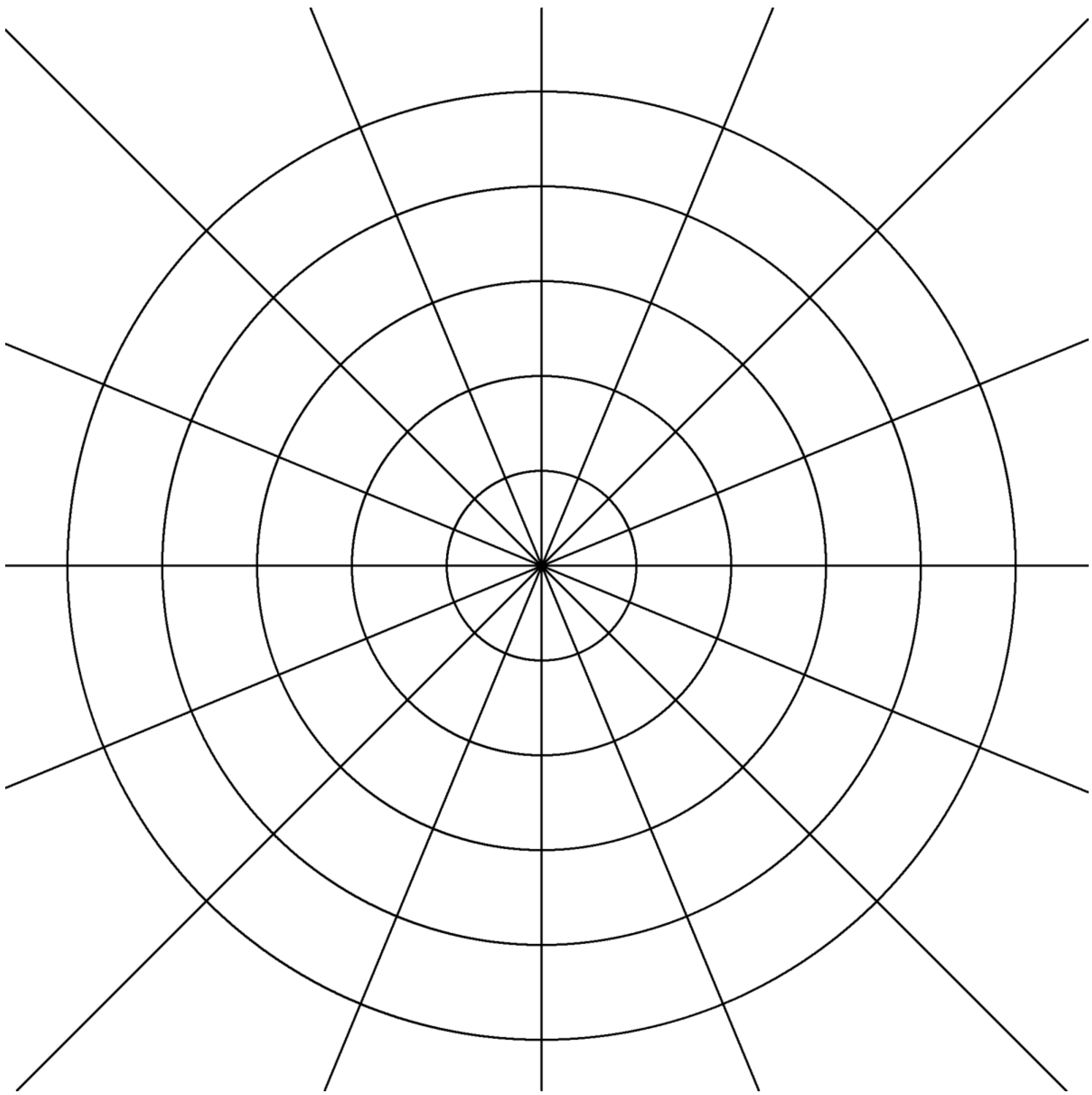
13 Polar Equations

$r=1 \quad r=2 \quad r=3 \quad r=4 \quad r=5$

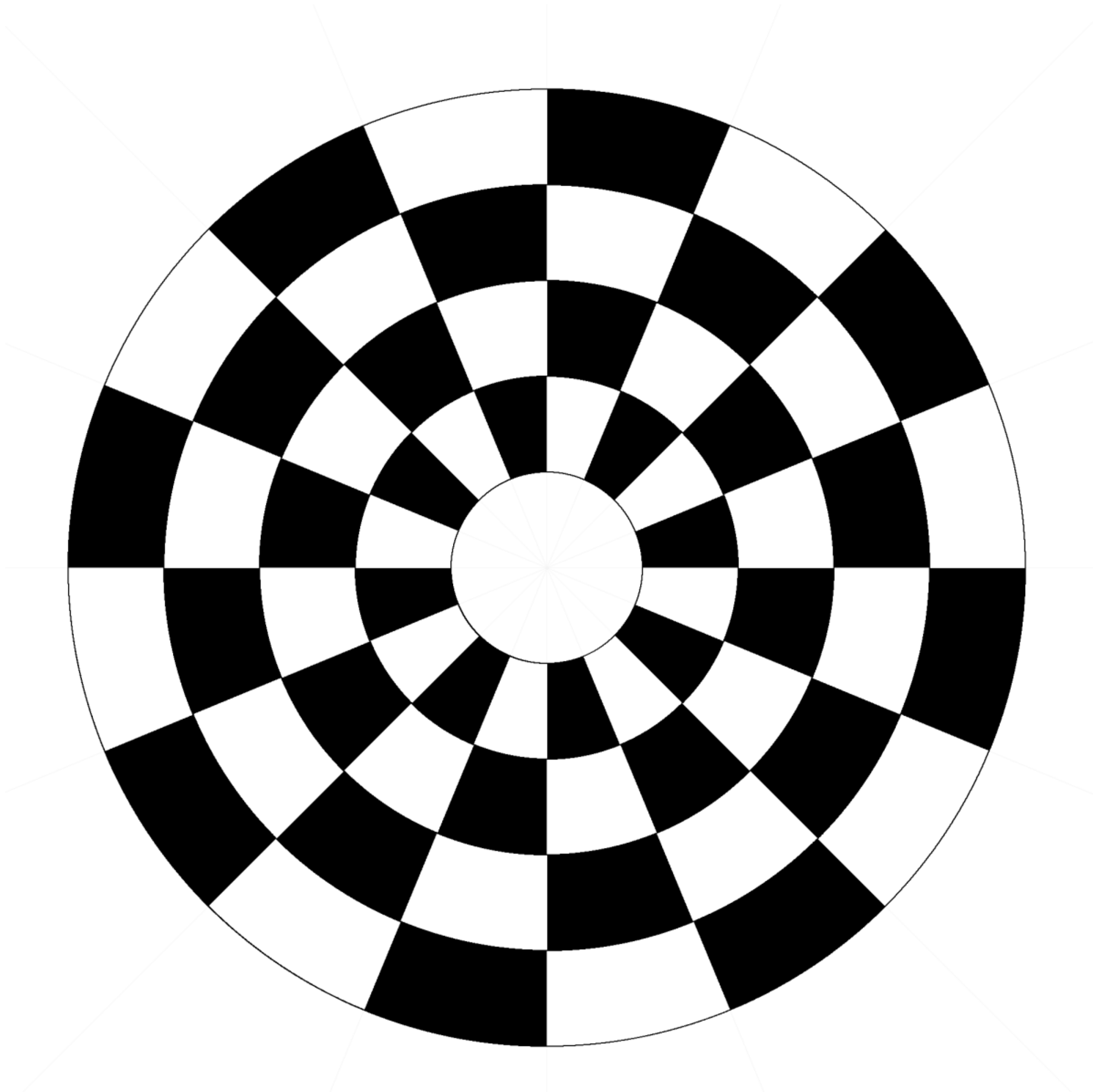
$\theta=0 \quad \theta=\frac{\pi}{8} \quad \theta=\frac{\pi}{4} \quad \theta=\frac{3\pi}{8}$

$\theta=\frac{\pi}{2} \quad \theta=\frac{5\pi}{8} \quad \theta=\frac{3\pi}{4} \quad \theta=\frac{7\pi}{8}$

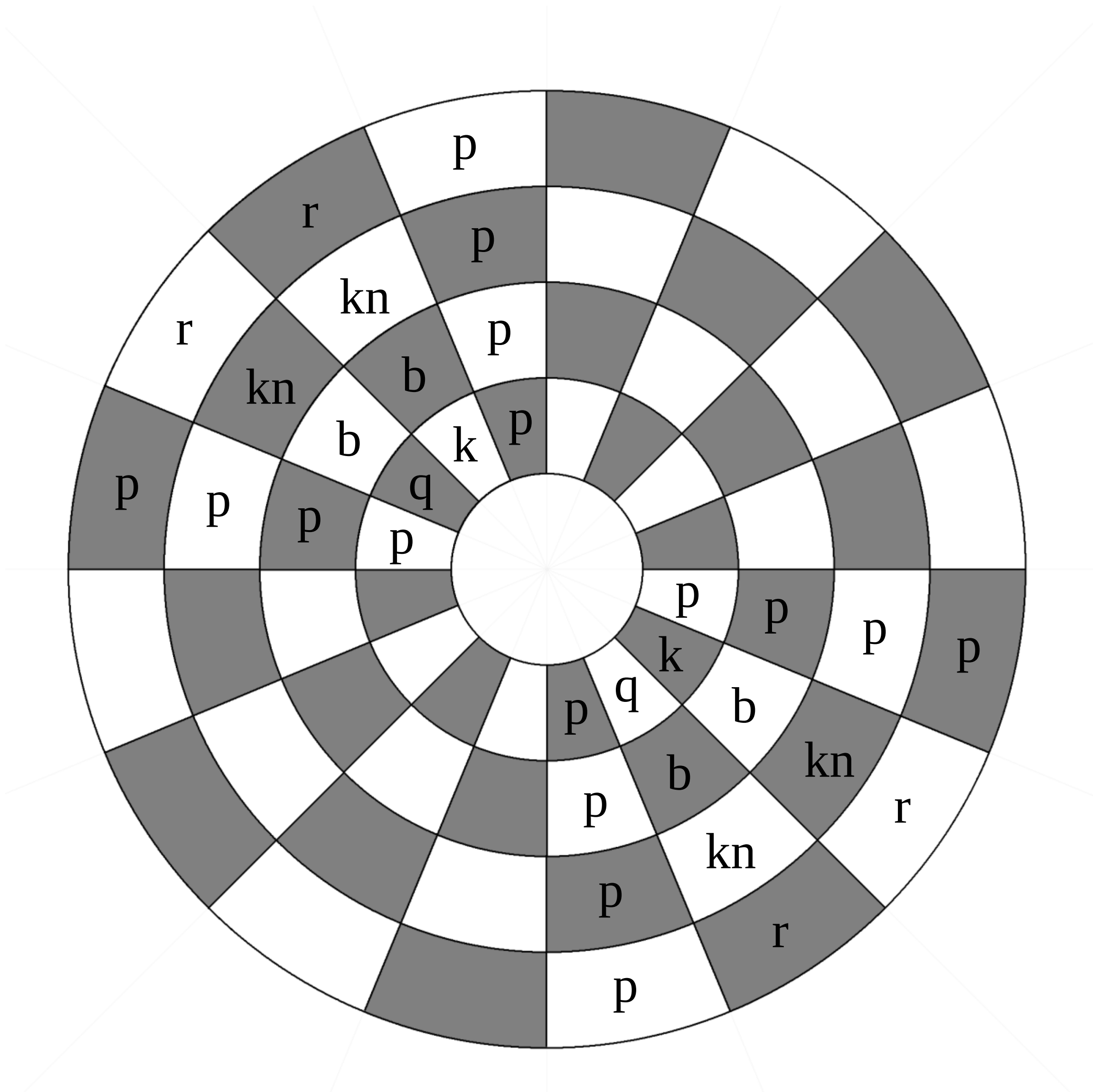
13 Polar Equations

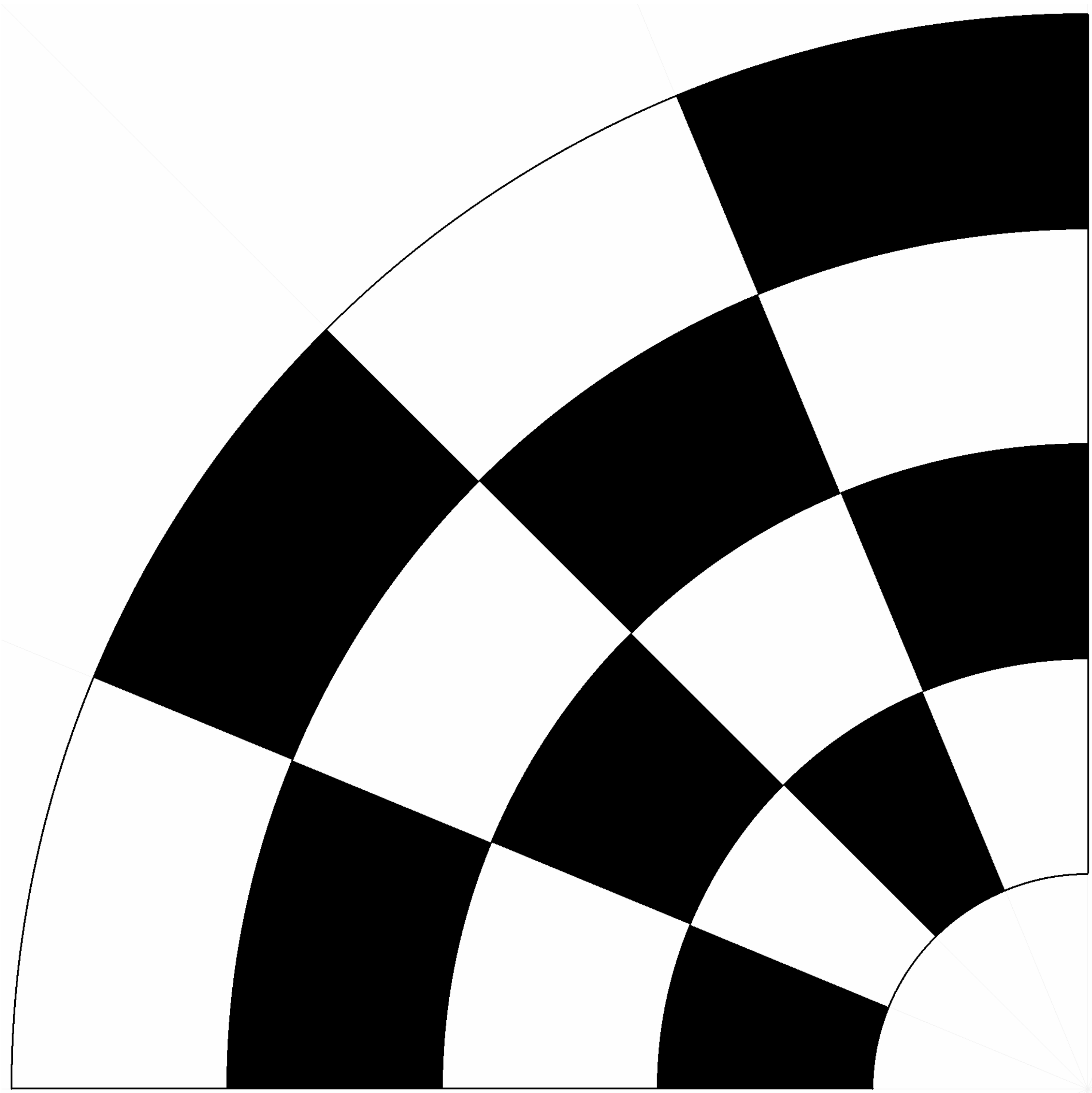


Circular Chess



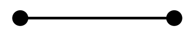
Circular Chess



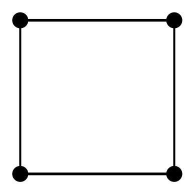


The *n*-cube

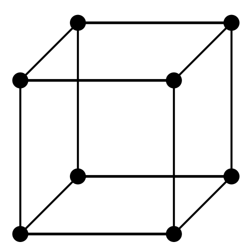
$n = 0$



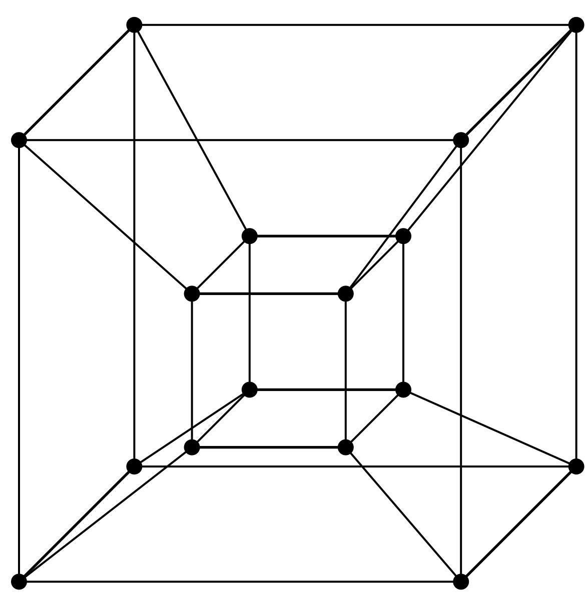
$n = 1$



$n = 2$

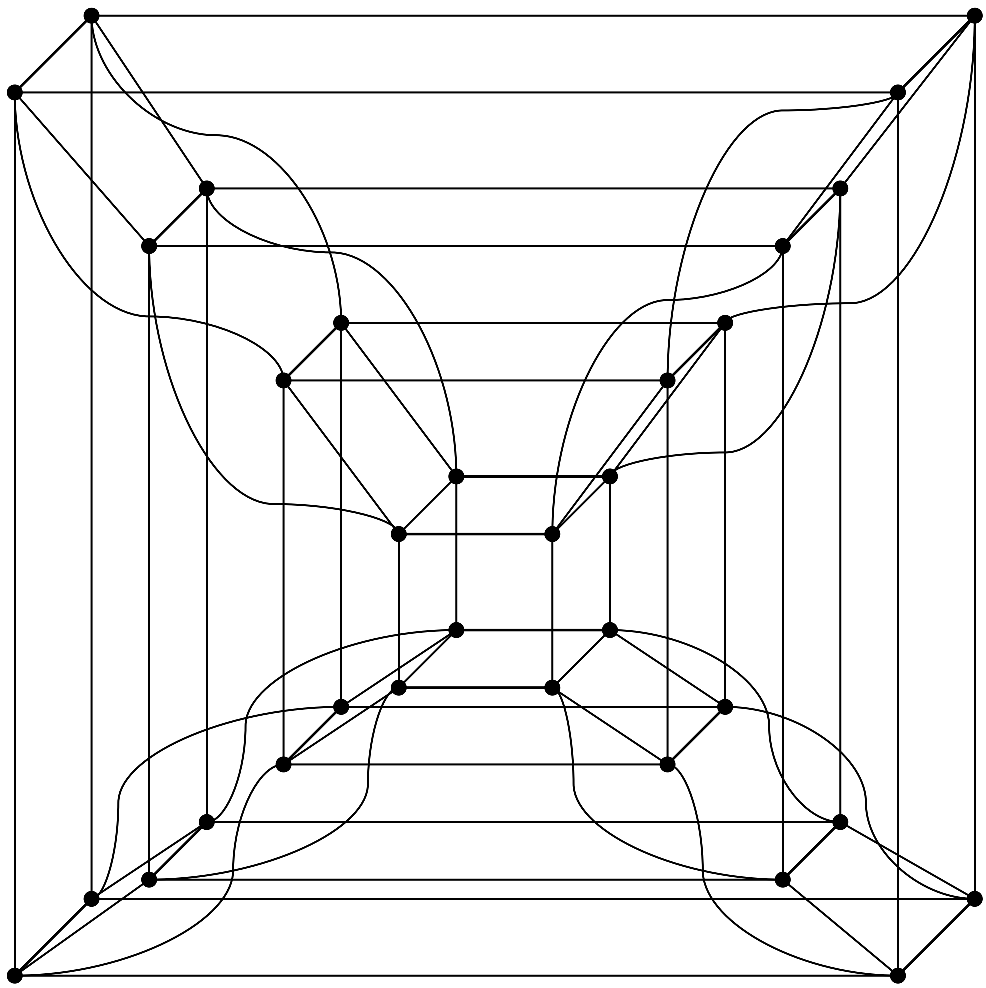


$n = 3$



$n = 4$

The 5-cube



Some *n*-cube Facts

An n-cube is also called an n-dimensional hypercube.

0-cube	point (vertex)
1-cube	line segment (edge)
2-cube	square
3-cube	cube
4-cube	tesseract

How many k-cubes are there in an n-cube? $\binom{n}{k}2^{n-k}$

The number of k-cubes in an n-cube can be represented using the following generating function.

$G_n(k) = (2 + k)^n$

$G_0(k) = 1$

$G_1(k) = 2 + k$

$G_2(k) = 4 + 4k + k^2$

$G_3(k) = 8 + 12k + 6k^2 + k^3$

$$G_4(k) = \underset{\text{vertices}}{\underset{\uparrow}{16}} + \underset{\text{edges}}{\underset{\uparrow}{32k}} + \underset{\text{squares}}{\underset{\uparrow}{24k^2}} + \underset{\text{cubes}}{\underset{\uparrow}{8k^3}} + \underset{\text{tesseracts}}{\underset{\uparrow}{k^4}}$$

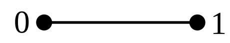
The 4-cube includes 16 vertices, 32 edges, 24 squares, 8 cubes, and 1 tesseract.

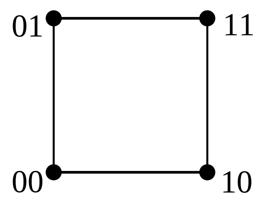
Labeling the *n*-cube: To label the n-cube, let each vertex be a bit string of length n. An edge connects two vertices that differ in exactly one bit.

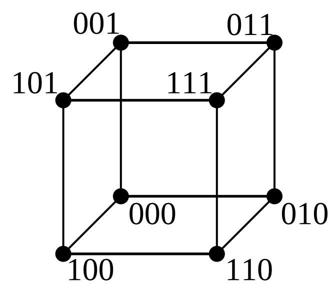
Graphing the *n*-cube: The weight of a bit string is the number of 1s in that bit string. To graph the n-cube, let each vertex of weight k be placed in level k. Edges only exist between consecutive levels. This creates a partially ordered set.

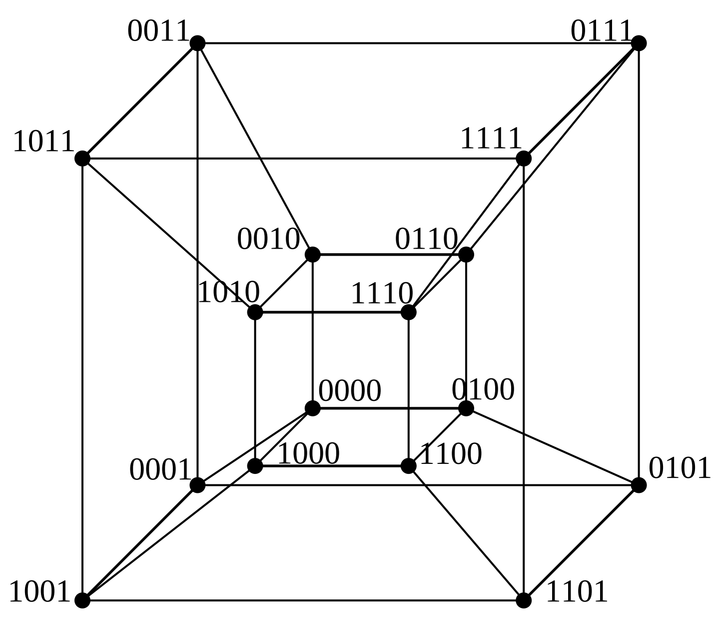
You can build the tesseract using Zome building sets.

Labeling the *n*-cube

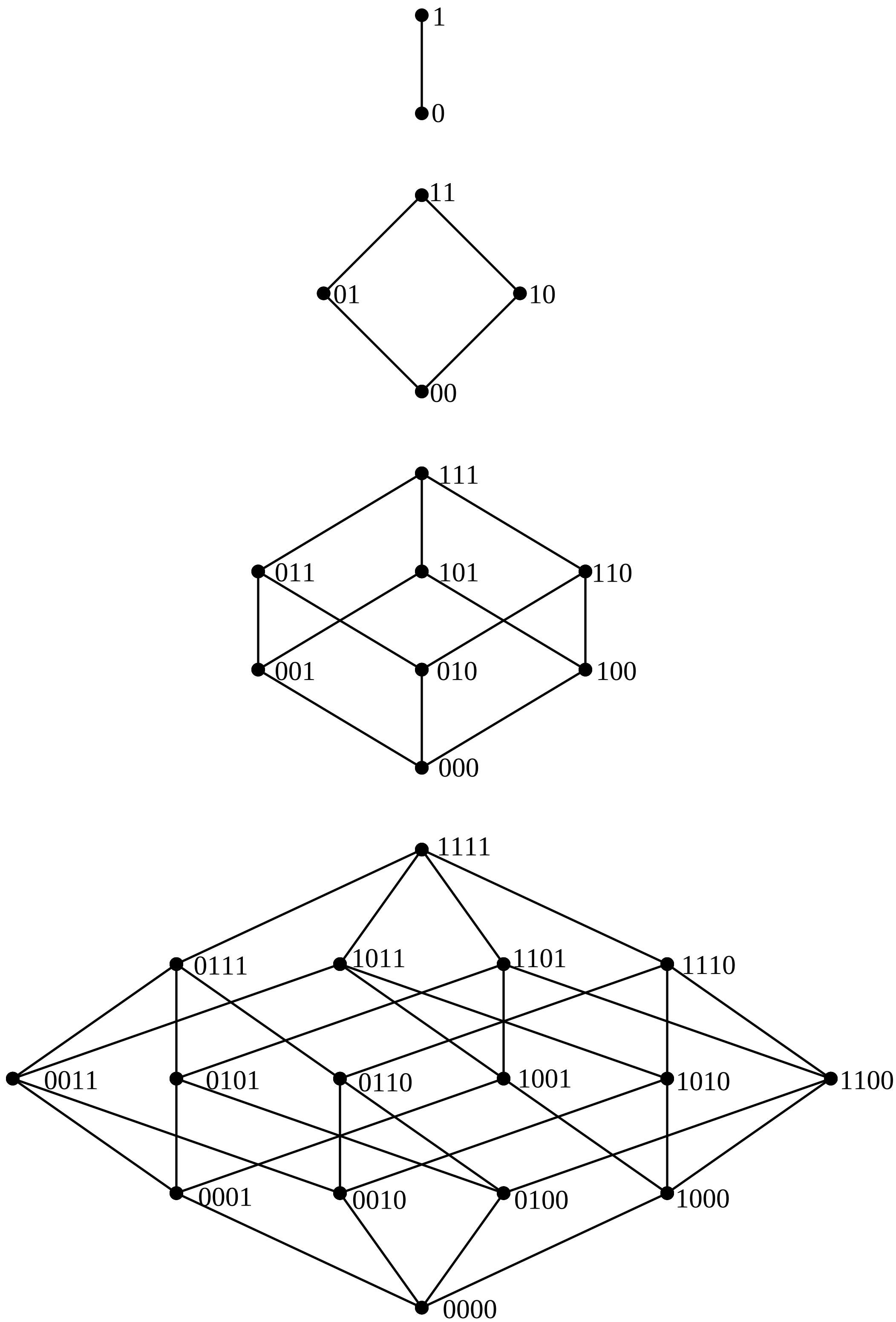




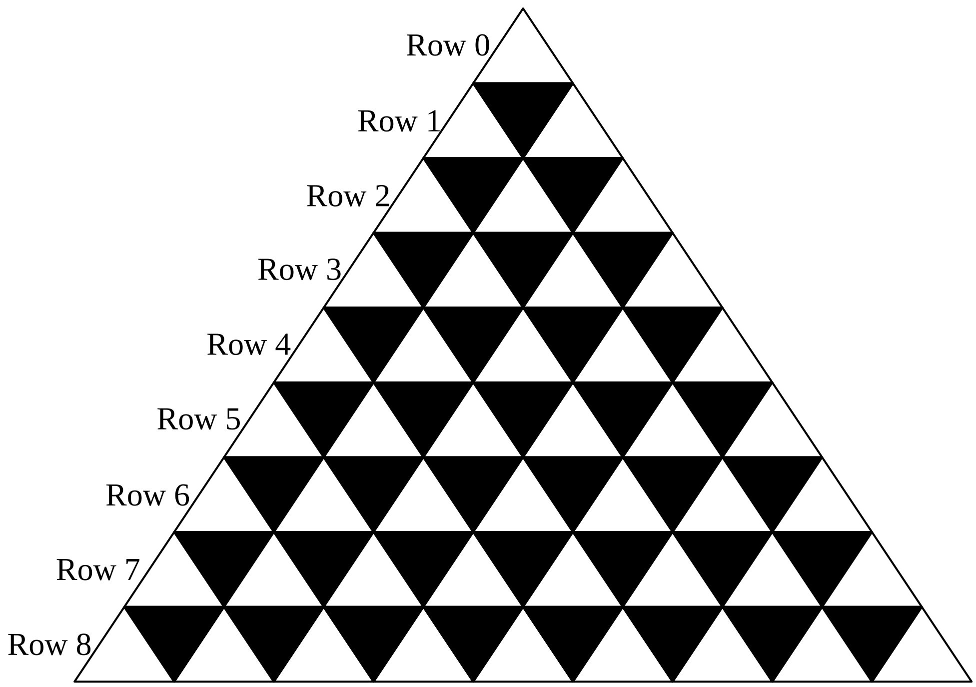




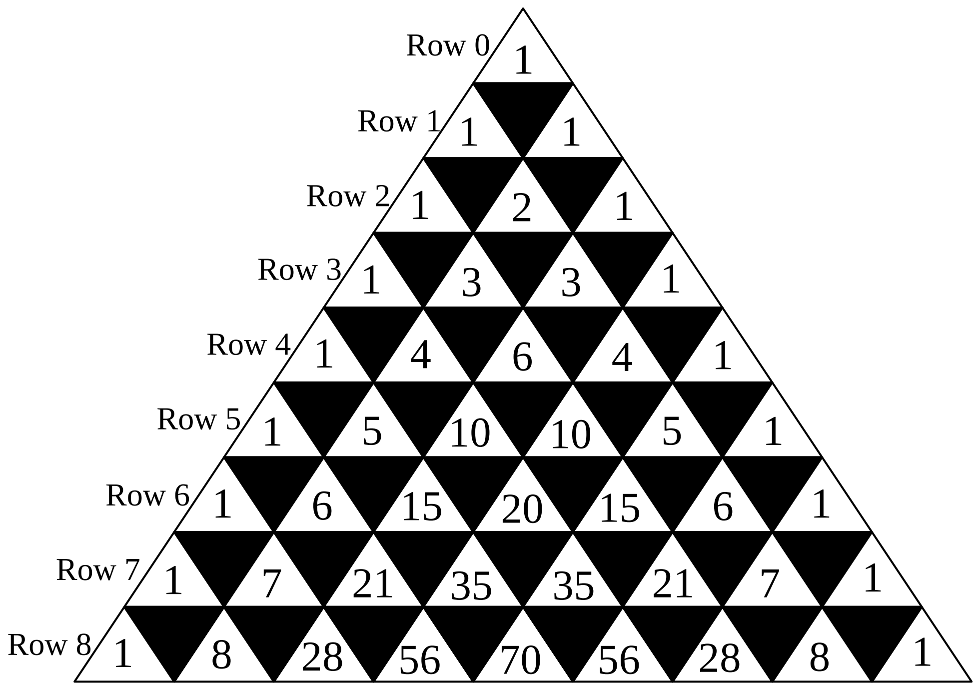
Graphing the *n*-cube



Pascal's Triangle

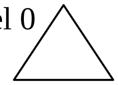


Pascal's Triangle

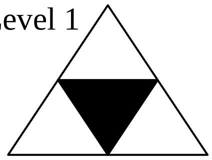


Pascal’s Tetrahedron

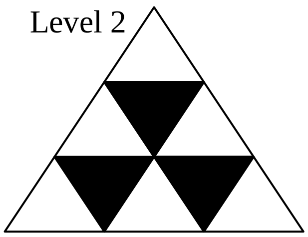
Level 0



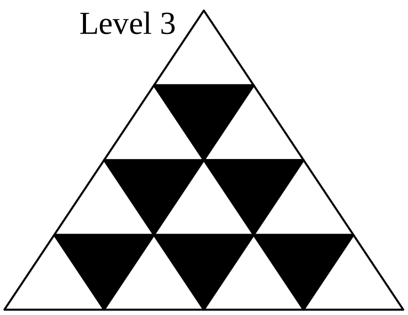
Level 1



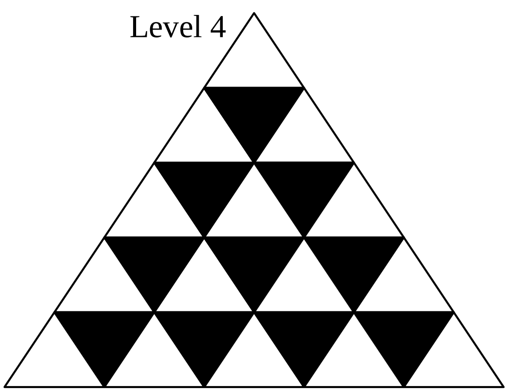
Level 2



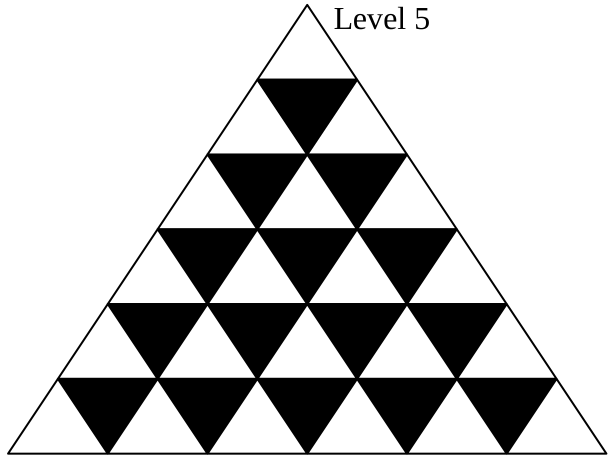
Level 3



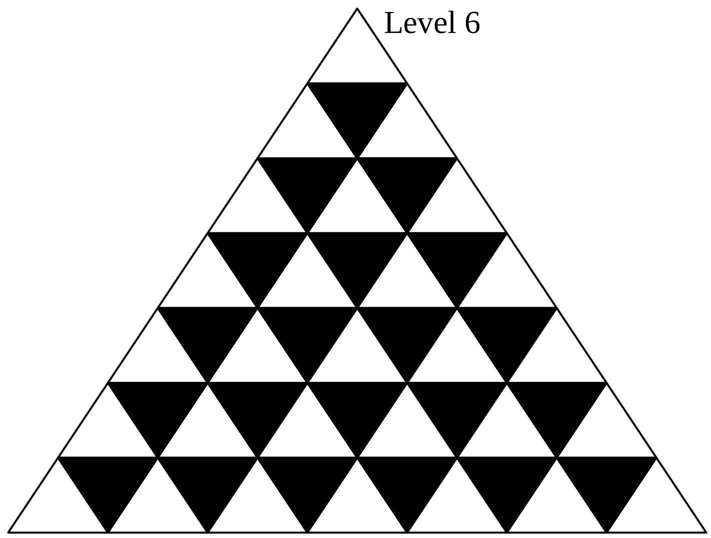
Level 4



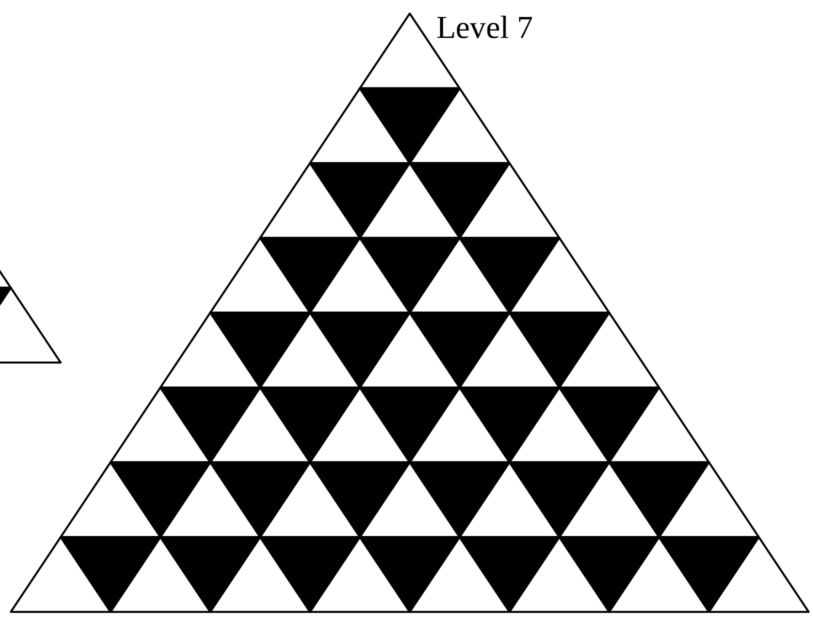
Level 5



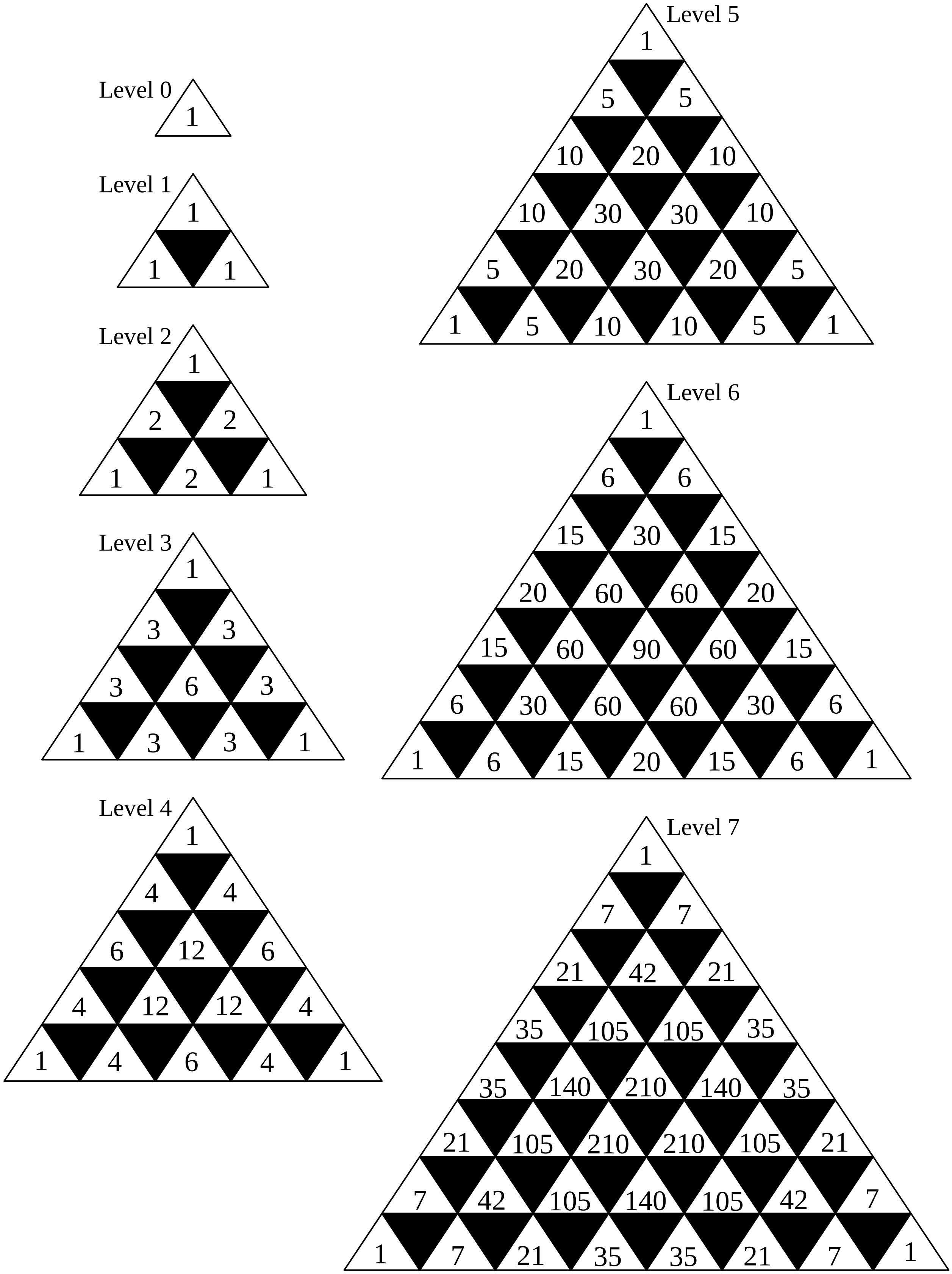
Level 6



Level 7



Pascal's Tetrahedron



Some Facts About Pascal's Tetrahedron

1. Pascal's triangle is seen on the three sides of Pascal's tetrahedron.

2. Pascal's tetrahedron has rotational symmetry.

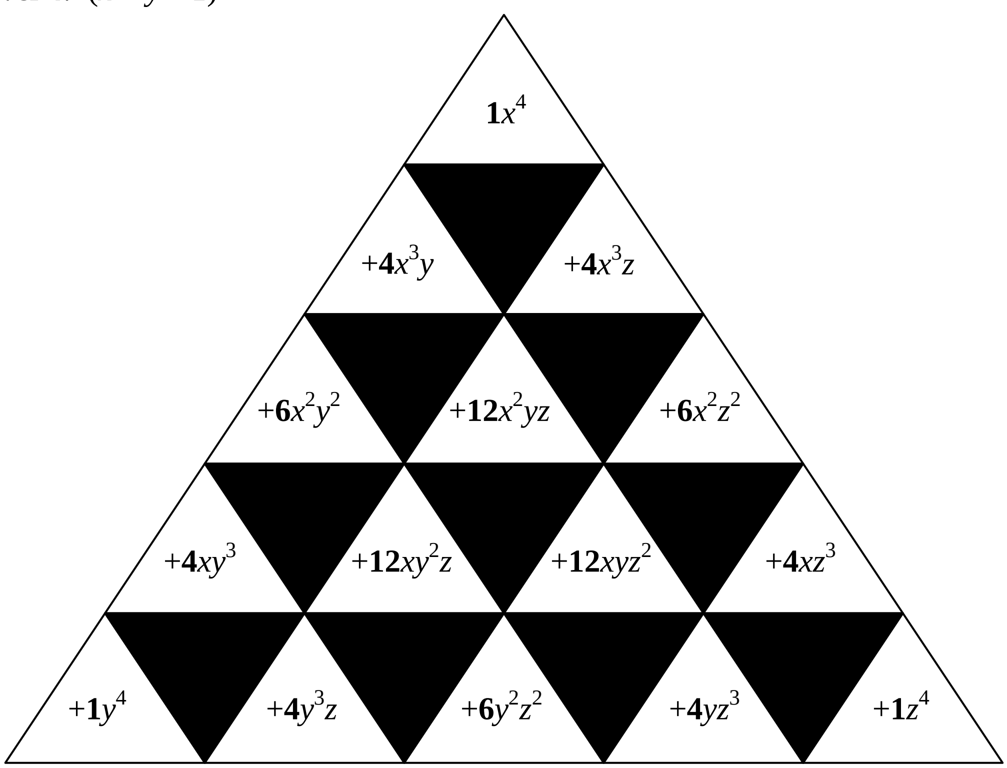
3. For Pascal's triangle, the sums of the entries in row n is 2^n.
   For Pascal's tetrahedron, the sum of the entries in level n is 3^n.

4. Pascal's triangle gives binomial coefficients.

   row 4: $(x + y)^4 = \mathbf{1}x^4 + \mathbf{4}x^3y + \mathbf{6}x^2y^2 + \mathbf{4}xy^3 + \mathbf{1}y^4$

   Pascal's tetrahedron gives trinomial coefficients.

   level 4: $(x + y + z)^4$



5. Pascal's triangle solves a variety of counting problems.

❶ Flip a coin 6 times. How many different ways can the outcome have 4 heads and 2 tails?

❷ How many different "words" can be formed using all the letters of the word *BOOBOO*?

❸ A family has 6 kids. A group of 4 is to rake leaves, and a group of 2 is to wash the car. How many ways can the groups be formed?

All three questions have the same answer. Go to row 6, point to the first 1, and move 4 to the right. The answer is 15 and can also be calculated by $\frac{6!}{4!2!}$.

Pascal's tetrahedron solves a variety of counting problems.

❶ A three-sided die is labeled A, B, C. Roll this die 6 times. How many different ways can the outcome have 3 As, 2 Bs, and 1 C?

❷ How many different "words" can be formed using all the letters of the word *BANANA*?

❸ A family has 6 kids. A group of 3 is to rake leaves, a group of 2 is to wash the car, and 1 is to pick up the dog poop. How many ways can the groups be formed?

All three questions have the same answer. Go to level 6, point to the top 1, move 3 to the southwest, and 2 to the southeast. The answer is 60 and can also be calculated by $\frac{6!}{3!2!1!}$.

6. For Pascal's triangle, the number of entries in row n is $n + 1$.

For Pascal's tetrahedron, the number of entries in level n is $\binom{n+2}{2}$, a triangular number.

7. The graphs of Pascal's triangle and tetrahedron are made up of vertices and edges.

The following table gives the amount of materials needed to create Pascal's triangle down to row n. Notice the triangular numbers.

row	0	1	2	3	4	5	6	n
vertices	1	3	6	10	15	21	28	$\binom{n+2}{2}$
edges	0	4	10	18	28	40	54	$n(n+3)$

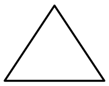
The following table gives the amount of materials needed to create Pascal's tetrahedron down to level n. Notice the tetrahedral numbers.

level	0	1	2	3	4	5	6	n
vertices	1	4	10	20	35	56	84	$\binom{n+3}{3}$
edges	0	3	12	30	60	105	168	$\frac{n(n+1)(n+2)}{2}$

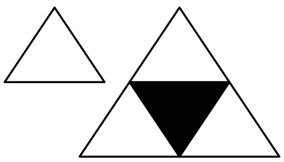
You can build Pascal's tetrahedron using Zome building sets.

Pascal's Hypertetrahedron

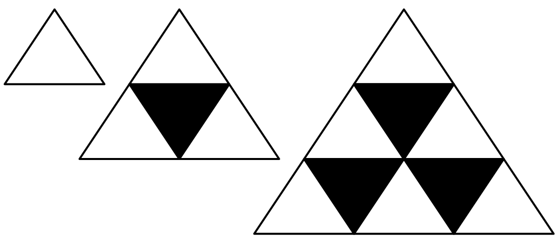
Level 0



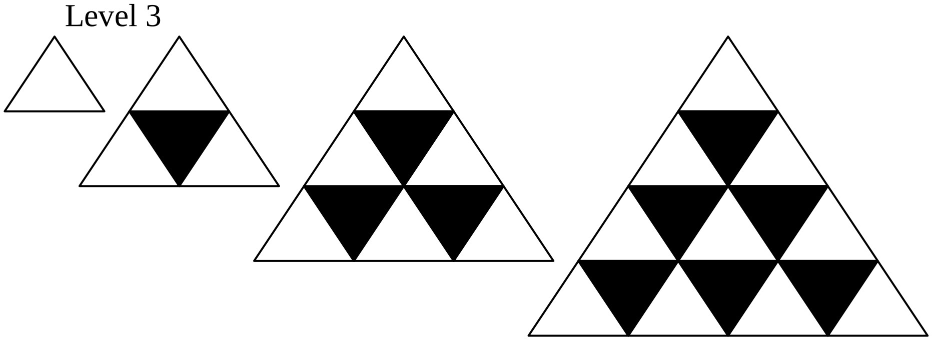
Level 1



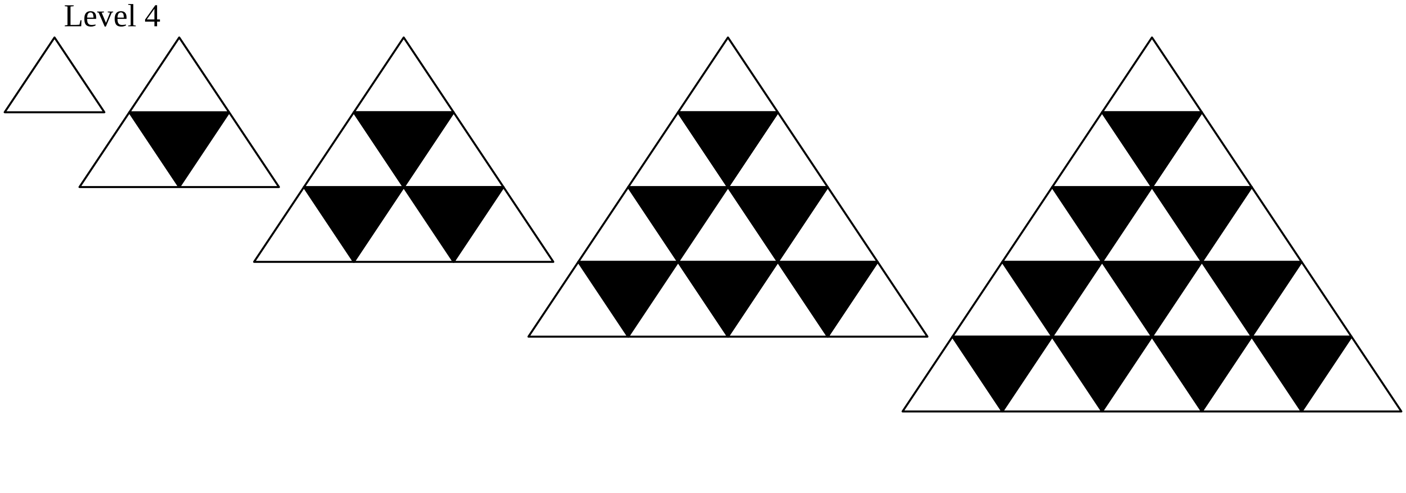
Level 2



Level 3

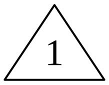


Level 4

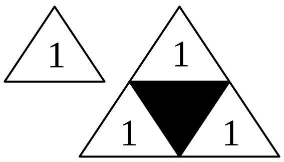


Pascal's Hypertetrahedron

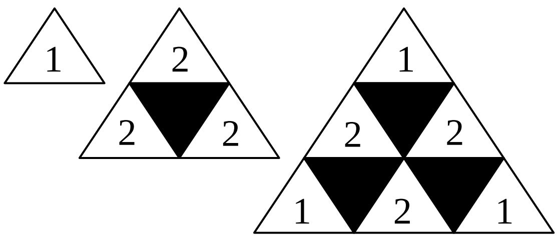
Level 0



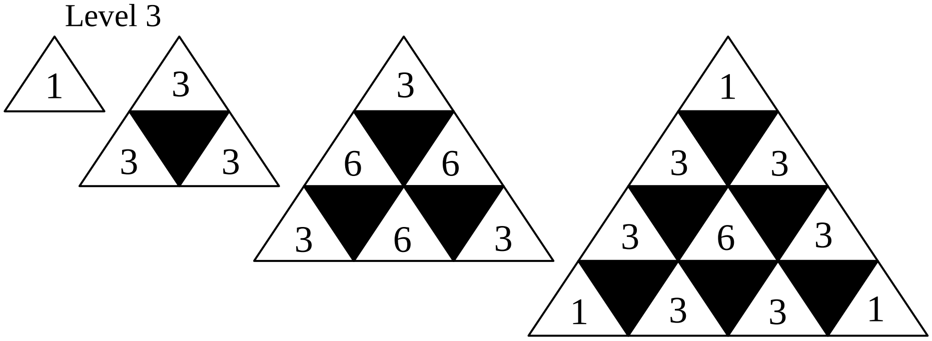
Level 1



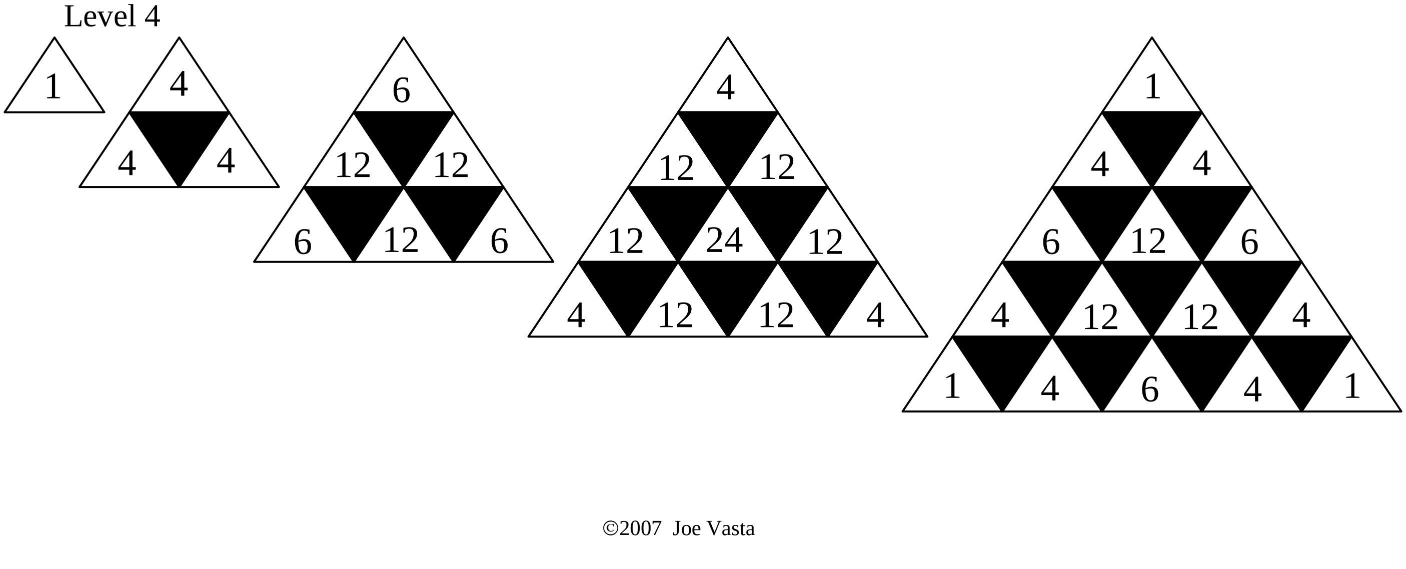
Level 2



Level 3

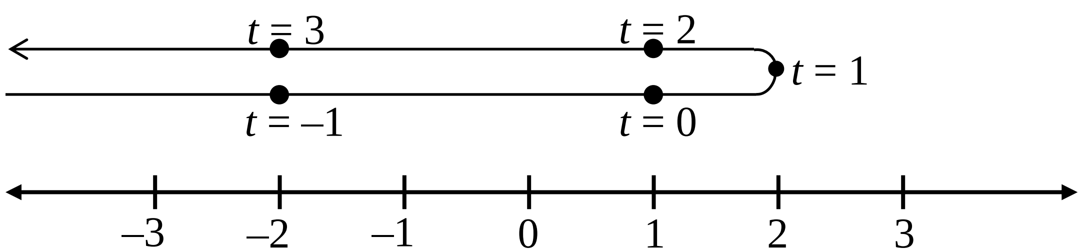


Level 4



Calculus Bug

Suppose a bug is moving on a number line. Let the position be measured in feet and the time be measured in seconds. Let [0, 3] be the time interval from 0 sec to 3 sec.



❶ displacement [0, 3]? –3 ft

❷ average velocity [0, 3]? –1 ft/sec

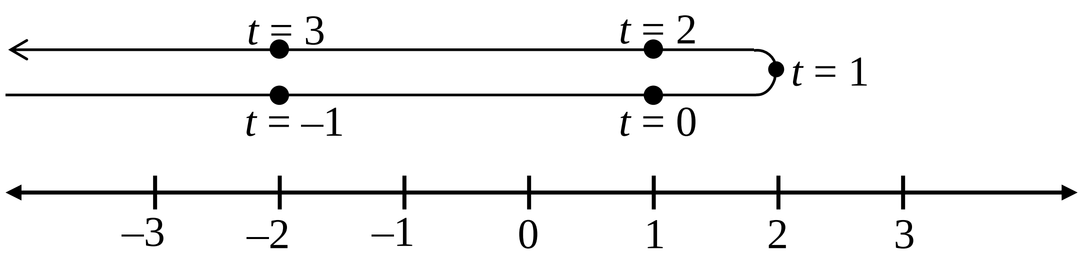
❸ velocity at 0 sec? It looks positive.

❹ resting time? 1 sec

❺ total distance traveled [0, 3]? 5 ft

● Derivatives are developed as the limits of the average velocities.

position function: $f(t) = -t^2 + 2t + 1$
velocity function: $v(t) = -2t + 2$



❶ disp. [0, 3]

$f(3) - f(0) = -3$ ft

❷ a.v. [0, 3]

$$\frac{f(3) - f(0)}{3 - 0} = -1 \text{ ft/sec}$$

❸ vel. at 0 sec

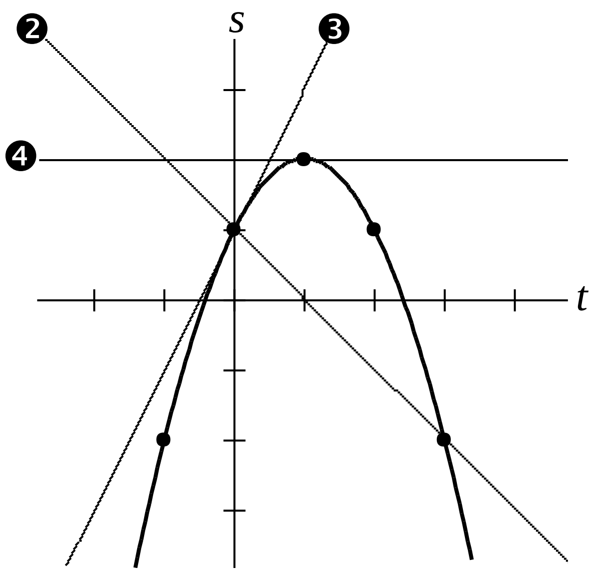
$v(0) = 2$ ft/sec

❹ resting time: Set $v(t)$ equal to zero.

$t = 1$ sec

❺ t.d.t. [0, 3]: Use the resting time.

$$\left|\text{disp.}[0, 1]\right| + \left|\text{disp.}[1, 3]\right| = 5 \text{ ft}$$



$$f(t) = -t^2 + 2t + 1$$

❶ disp. [0, 3]

$f(3) - f(0) = -3$ ft = displacement on the *s*-axis

❷ a.v. [0, 3]

$\dfrac{f(3) - f(0)}{3 - 0} = -1$ ft/sec = the slope of the secant line

❸ vel. at 0 sec

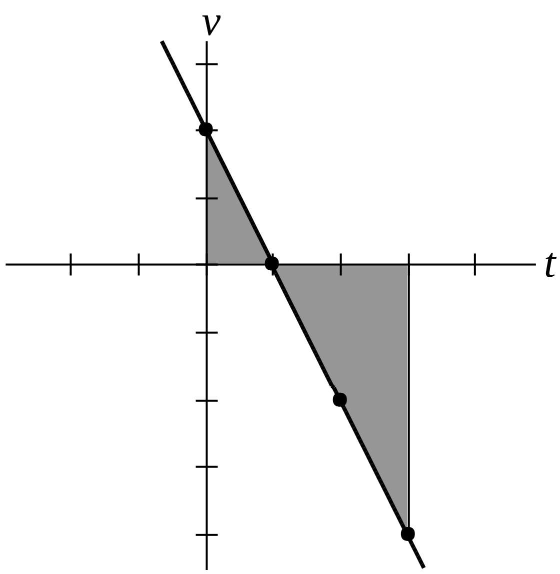
$v(0) = 2$ ft/sec = the slope of the tangent line at $t = 0$

❹ resting time

$t = 1$ sec, which is when the slope of the tangent line is 0

❺ t.d.t. [0, 3]

$\left|\text{disp.}\left[0,1\right]\right| + \left|\text{disp.}\left[1,3\right]\right| = 5$ ft = total movement on the *s*-axis



$$v(t) = -2t + 2$$

❶ disp. [0, 3]

$\int_0^3 v(t)\,dt = f(3) - f(0) = -3 \text{ ft} = \text{net area}$

❷ a.v. [0, 3]

$\frac{1}{3-0}\int_0^3 v(t)\,dt = \frac{f(3) - f(0)}{3-0} = -1 \text{ ft/sec} = v_{ave}$

❸ vel. at 0 sec

$v(0) = 2$ ft/sec = the value of the function at $t = 0$

❹ resting time

$t = 1$ sec = the t-intercept

❺ t.d.t. [0, 3]

$\left|\int_0^1 v(t)\,dt\right| + \left|\int_1^3 v(t)\,dt\right| = \left|\text{disp.}[0,1]\right| + \left|\text{disp.}[1,3]\right| = 5 \text{ ft} = \text{total area}$

- There are 16 Bug handouts.

- Polynomials are the only functions used (mostly).

- Covers speed, acceleration, & Mean Value Theorem.

- Higher dimensions are included with the last 3 handouts.

The Big X

Factor.

$6x^2 - 13x - 15$

Step 1 – Multiply 6 by 15.

$6x^2 - 13x - 15$ 90

Step 2 – List factor pairs of 90.

$6x^2 - 13x - 15$

90
1, 90
2, 45
3, 30
5, 18
6, 15
9, 10

Step 3 – Circle the last sign.

$6x^2 - 13x - 15$

90
1, 90
2, 45
3, 30
5, 18
6, 15
9, 10

Step 4 – Find the factor pair that subtracts (−) to give 13.
This step tells us whether or not we can factor the polynomial.

$6x^2 - 13x - 15$

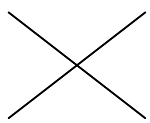
90
1, 90
2, 45
3, 30
5, 18
6, 15
9, 10

Step 5 – List factor pairs of 6. List factor pairs of 15.

$6x^2 - 13x - 15$		90
1,6	1,15	1, 90
2,3	3,5	2, 45
		3, 30
		5, 18
		6, 15
		9, 10

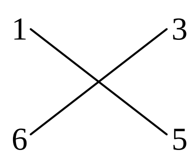
Step 6 – Draw a big X.

$6x^2 - 13x - 15$		90
1,6	1,15	1, 90
2,3	3,5	2, 45
		3, 30
		5, 18
		6, 15
		9, 10



Step 7 – Put appropriate factor pairs on the big X so that multiplication across the two lines gives 5 and 18. How can you do this? Start with 5. What two natural numbers will give a product 5? Only 1 and 5. Do you see any factor pairs (under the big X) that have a 5? Only one factor pair: 3 and 5. You must go with this one. Circle it and put 3 and 5 on the big X directly below (in any order). Look at the 5 you just wrote down. Travel across the line segment (northwest) and put 1 (since 5 times 1 gives you 5). Now circle the 1 and 6. The 6 goes on the last empty spot on the big X. Travel across the line segment from 6 (northeast) and you will see 3 (6 times 3 gives you 18). Done. This step should not be "guess and check" if you really think at the beginning.

$6x^2 - 13x - 15$		90
1,6	1,15	1, 90
2,3	3,5	2, 45
		3, 30
		5, 18
		6, 15
		9, 10



Step 8 – Look at the original polynomial. The middle term's coefficient is –13.
Put signs on your original factor pair so they add to –13.

$6x^2 - 13x - 15$

1,6	1,15		90
2,3	3,5		1, 90
			2, 45
			3, 30
1	3		+5, –18
			6, 15
6	5		9, 10

Step 9 – Put corresponding signs near the numbers on the right side of the big X.

$6x^2 - 13x - 15$

1,6	1,15		90
2,3	3,5		1, 90
			2, 45
			3, 30
1	– 3		+5, –18
			6, 15
6	+ 5		9, 10

Step 10 – Circle the top numbers and circle the bottom numbers on the big X.
Write your answer.

$6x^2 - 13x - 15$

1,6	1,15		90
2,3	3,5		1, 90
			2, 45
			3, 30
1	– 3		+5, –18
			6, 15
6	+ 5		9, 10

Answer: $(x-3)(6x+5)$

After doing the big X many times, you may find yourself skipping some or all of the steps on certain problems. That just means you are getting good at factoring trinomials.